Reasoning in Distributed Parameter Systems – Migrating Object Approach

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Abstract

In the paper some problems of distributed reasoning are considered from the point of view of object oriented methods. The time-space relationships are approached both as the partial differential equations and special maps constructed to model local phenomena. Canonical objects-agents are formulated for observing the space of distributed parameter systems. Migrating canonical agents are specified to enable process recognition when the system is considered to be open. Definition of openness is given as the bridging of two frameworks or open boundary in the system. Invariant measure chosen for migrating agents is defined to identify local properties and choose unique value representing local behaviour. Examples of applications of the method are shown.

1. Introduction

We consider parameter distributed systems. In such systems, as the parameters change boundaries assumed for the model may become not valid, hyperdifferentiation in degrees of freedom, values and dimensions can grow. However, the model has to retain its cognitive properties. The reasoning of this paper is based on the assumption that the global properties and parameters are unlearnable. It is therefore important to observe that standard arguments as well as the putative de-differentiating properties at the boundaries must be learned. Very frequently it turns out to be very confusing to draw conclusions from lower order (and lower dimension) models that are used to represent higher order structures.

In the paper de-differentiation is proposed to be performed by new entities – migrating canonical agents. Canonical agents work as cognitive bookmarks and let emerge complex local contexts. The important observation is that local context may not be properly described by global model domain thus introduced new notions are formulated in order to build universal data networks.

In theoretical considerations of distributed parameter systems we think about communication, information, flow of information. The cyborg point of view must be literal, material or technical; it is built or developed, located and specific and as such must be incorporated in the cognition ([5], [6], [7], [10]). If the cyborg is controlled the controls can be in "equilibrium" whereas the state may develop. In other cases specific state is only found by deriving complex formulas for the control variables. Thus, the appropriate measures need to be defined for the recognition of dynamics, possibly locally in space or in time changing its qualitative properties. The principles based on relevant measures of uncertainty are fundamental to reasoning when the global and local properties are considered and symmetries are locally broken. These can be the principles of Minimum Uncertainty, Maximum Uncertainty, and Uncertainty Invariance. In the work we focus our attention by choosing the reasoning based on the Principle of Uncertainty Invariance which is formulated as follows. This principle requires that the amount of uncertainty be preserved when we transfer uncertainty formalised in one category into an equivalent formalisation in another category. The representation of above principle considered in this paper has the form of minimum information loss (minimum of entropy production).

In the formulation of the system strategic function, operational function, relationships, conceptualisation, methodology, elements and openness must be defined for the system to be effective. The unified methodology is assumed to preserve the meaning of basic definitions regardless of the strategic function. We are interested in non-linear problems. In highly non-linear problems the convergence properties of approximation may not be shown. But for some specifically chosen approximating probe functions their properties can be analysed and used in local consideration. Thus, after having the global goals of recognition or control formulated, the core concepts of operational function are chosen.

They are depicted by:

- Dynamic means of the phenomena interpretation – they describe framing of the problem;
- Learning of the system - particular domain of learning must be named e.g. it can be considered as a network of objects with possible new links to be fitted and accommodated; some results from neural network theory can be used.

There are several basic questions that must be considered: what are the elements of modelling concepts that will be used for describing associative interactions. The structure of derived relationships indicates that neural network concepts can be applied to model associativity and configuration. The operators in the canonical agents, which make the concepts of association and configuration, are formulated in a direct way and use basic methodology of neural networks. However, the results from neural networks theory are not applicable here, as some models known to the author do not confirm them. To give an example the Fick's model describing diffusion cannot be trained. The next question is, what is the definition of complementarity and what higher level structures can be unfolded due to complementarity? The operators, which make complementarity, are assumed to be derived from detailed investigation of the particular phenomena considered, its limit properties and sensitivity analyses. Distributed properties of the system can be in many cases incorporated in its limit form by solving optimisation problems, by qualitative solutions of subsystems or discrete recursive algorithm. Complementarity in the set of directions and problems introduced is essential in cognition. Similarity in categorisation for database retrieval was the typical approach.

The method of recognising a new is crucial in the cognition process. In our approach the concept new is considered to represent:

- The properties of bridged dynamics that describe local phenomena.
- Open boundaries: e.g. in diffusion processes intermediate distance is preferred instead of the closest strong ties; in language related recognition the strong open boundary might be explored by testing natural languages; in neural networks it is based on the "spreading activation" approach.

2. Data structure specification

The ideal environment of storing and linking canonical agents is a hypertext, because it can be conceptualised and may easily be scanned and browsed. The expert system black-box that is an "early-warning automaton" and does not consider changed qualitative properties of the process is not applicable. In our system, with possible physics, biological element, human behaviour in the loop, the concept of a text – metaphor is introduced. Hypertext works as an association machine that tries to understand or amplify the potential of the internal element by communication. The global function and operational function interaction is introduced at nodes.

The structure of a hypertext encompasses the following elements:

- Canonical agents form the nodes in the hypertext.
- Links between canonical agents form directions.
- Sets of interconnected canonical objects + methods of solutions form problems.
- Chains of links form clusters or complex category (new context).

3. Unified analytical methodology

If the general notion of canonical agent is formulated in the high-level description then the methods of one area can be applied in another. A specific user might be interested in more complex level of abstraction with physics-in-the-loop strategy. Sources of high-level canonical agents can be identified from models of local nonlinearities, numerical approximations of non-linear problems, chaotic maps, quantum physics, quantum mechanics, quantum computing methods derivations and neural networks modelling. In the work the canonical objects are derived in complete form for non-linear systems. The assumption is that characteristics carry sufficient information to classify the models and derive complementarity conditions. In order to define canonical objects we need model operators. The original models must be expressed in terms of the new operators formulated in canonical objects. This procedure demands detailed investigation and construction of computation of non-commuting operators. The commutation properties are of particular importance for two operators: the constructor operator of the canonical object and the action law operator of the phenomena. Using the concept of characteristics performs the classification and initiation of a hypertext. Information encompassed by the characteristics is large enough for global performance analysis, control and for the insight into local or limit properties. Given the consideration which must combine mathematical approach with incorporated physical, technical or human system the recognition must be viewed both as a problem with respect to small parameter and with respect to smoothness of the state variables. The problem of constructing converging behaviour with respect to changing parameter can be regarded in its general approach as the following: for a given operator in the appropriate scale of the spaces an almost inverse operator should be constructed.

We choose a canonical atlas A of a manifold M and $\{U_j\}_{j=1,\dots,K}$, $\{U'_j\}_{j=1,\dots,K}$ as maps of the atlas A which cover M. Algebra is equipped in μ – structure that satisfies necessary axioms:
For the maps U_j, U'_j two axioms are required:

V1 – the locality of operator $V_{U,U'}$ which transforms any function φ from U to U' and its value depends only on variation of φ

V2 - consistency condition $[V_{U1,U'1}](x_0) = [V_{U2,U'2}](x_0)$ for any φ with a support in a projection of $U_1 \cap U_2$ on appropriate parameter interval.

4. Case study

As an example non-linear system is considered and the method is applied for singularity tracking. In a strict mathematical sense the considered example phenomenon is supposed to be governed by a set of hyperbolic non-linear partial differential equations.

In our approach when the conditions for global existence are not met we are thinking about viscous solutions, where new definitions must be introduced in order to approximate non-linear terms by chosen probe functions.

The point about such equations is that even for smooth initial data the solution surface may fold up and develop an infinite derivative in finite time. In order to continue the solutions through the development of such singularities methods of making a proper local choice must be constructed. In particular one might be interested in the performance in the neighbourhood of singularity. An instantiation of migrating object is conceptualised and derived for singularity tracking.

5. One dimensional non-linear hyperbolic system

The derivation of unified method is supported by many examples. Let us consider one-dimensional case.

We write the equation

$$u_t + f(u)_x = 0 \qquad (1)$$

in the following form, along the characteristics where we must have u = constant.

$$x = s, \text{ when } t = 0;$$

$$u = u_0(s);$$

$$dx/dt = f_u(u_0(s)) \text{ when } t > 0$$

or alternatively, for a given $u_0(s)$

$$u(x,t) = u_0(s), \qquad (2)$$

where $x = t\, f_u\,(u_0(s)) + s$.

The equation (2) is a solution of (1) provided that u_t and u_x remain finite. When the u_t and u_x are not finite the condition for the instantiation of a singularity tracking migrating canonical object-agent is of the form:

$$t d/dt\, [\, f_u(u_0(s))\,] + 1 = 0, \qquad (3)$$

where $u_0(s)$ is a parameterised solution of (1).

In the neighbourhood of the developed shock qualitatively new entity can be considered. New entity satisfies complementarity definition required in our system. New entity approximates diffusion phenomenon. When the condition of singularity appearance is met the reasoning is based on density distribution in the open object. Chaotic map is introduced to model the diffusion in the system

6. Approximation of the local choice

In order to find the unique local solution extracted by migrating object the criterion of minimum loss of information is assumed. For the extracted objects a special measure is derived. As an example the constructing procedure considered here follows the work of Gaspard ([3], [4]). The measure of information is an entropy production measure. The chosen analytical method of reasoning is derived in a strict mathematical form and is based on the flow analysis.

The system is an open systems for which we have to define a measure describing local behaviour of the phase space. Local is understood in a sense of extracted (filtered) object and complementarity conditions applied to the object. This object must be considered in a new space, possibly constructed for the local phenomena. The implementation might be developed as a complementarity condition in a numerical method applied to the system or a parameterised system call. In particular, in our example extracted diffusive object can be described with the use of new measure formulated for systems at singularity. The constructed measure is not normalisable because it becomes singular in some direction. However, a stationary gradient can be maintained in the system.

The solution of equations modelling the system requires boundary conditions at the borders of the phase space. Such boundary conditions allow us to express the non-equilibrium conditions in the following formulation: if the flow changes at the input and the output of the system, the corresponding boundary of phase space must be taken as a distribution with varying parameter. Non-flux or absorbing boundary conditions can be also considered to solve the equations. In the considered case we take into account flux boundary conditions. The system is assumed to be controlled. The flux boundary conditions give us a proper derivation to identify and use the entropy production as a measure of information loss in computational system.

The map used in derivation is a baker type map which is area preserving and of hyperbolic type. With the use of this map we consider a time evolution of a finite chain of squares $\varepsilon = (\Delta x, \Delta y)$. In a limit singularity must be approximated thus a singular measure is needed. The open boundary is understood in our reasoning as a hidden layer of description. As it was shown in ([7]) the local evolution is approximated by diffusive model. The cumulative functions are defined for the chain of squares. They exist even if the singularity occurs. These functions comprise the average flow in the object. Takagi incomplete functions which are differentiable almost everywhere define the cumulative functions. The incomplete Takagi functions converge to the Takagi function that is non-differentiable. Thus the results, which are needed, can be obtained due to the construction of

functions that converge to non-differentiable function. The ε - entropy production has the behaviour expected for diffusion.
The fourth order terms expressing entropy production are positive definite outside very narrow transition section. The minimum of derived entropy production criterion is found and it constitutes a unique local choice.

For large chains in the constructed measure the limit $L \to \infty$ (along the chain labels) is taken before the limit $\varepsilon \to 0$ (granularity of cells). Thus, the result shows an emerging property in a large system where entropy production does not vanish even in the fine scale limit. Large system is not understood in the sense of dimensionality. It is understood as a system with many degrees of freedom.

7. Conclusions.

The minimum entropy production criterion calculated in such way can be used as a complementary condition to ensure uniqueness of weak solutions in a locally emerging diffusion. In our considerations the criterion is applied for local information – retrieval when the result is needed as one value only. The theory predicts local future state only as probabilities. Computational system constructed generates the "unique local choice" of a variable to satisfy the global conditions.

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