Scientific Rationality and Inconsistency: A Partial Structures Perspective

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Abstract: *Traditional approaches to scientific rationality presuppose that consistency is a necessary condition for rationality. Although the requirement is sensible (in particular in the presence of classical logic), it fails to provide an adequate account of scientific practice, given the widespread existence of inconsistent scientific theories. Unless we are willing to entertain the idea that science is an irrational enterprise, we need to provide a model in which inconsistencies can be reconciled with rationality. In this paper I suggest an alternative picture of scientific rationality, one that is able to accommodate inconsistencies without triviality. The proposal employs a framework developed by da Costa and French in terms of partial structures and quasi-truth. Although inconsistent theories typically cannot be simultaneously true, they still can be quasi-true at the same time. Triviality is then avoided by the fact that the logic of quasi-true is paraconsistent. In this way, there is room for rationality even in the presence of inconsistencies.*
Keywords: rationality, inconsistency, paraconsistent logic, partial structures, quasi-truth.

1 Introduction

The traditional approach to scientific rationality presupposes that consistency is a necessary condition for rational theory change in science (see, for example, [13] or [10]). This presupposition has been adopted for an obvious reason: given the assumption that classical logic is the underlying logic of scientific theories, the inconsistency of a particular theory immediately establishes its triviality. That is, presupposing classical logic, if a given theory is shown to be inconsistent, then every sentence in the theory's language is true.

But this traditional approach faces a serious difficulty. The fact that inconsistencies have often been entertained in science (as well as in mathematics) makes this approach to scientific rationality quite unlikely. After all, according to this approach, substantial and significant parts of scientific practice are turn into pure irrationality.

Faced with this difficulty, some proposals have advanced more lenient criteria for rationality. Lakatos and Feyerabend, for example, have famously advocated accounts in which inconsistencies *can* be tolerated in science (see [11] and [9]). However, none of them provided a systematic framework in terms of which such tolerance could be properly implemented (and Feyerabend would be radically opposed to such an undertaking, given his own attitude toward rationality in general).

Moreover, in both views the acceptance of inconsistencies is associated with a type of "triviality": both in Lakatos's and in Feyerabend's approaches *anything goes*! Of course, in Feyerabend's case, this outcome is intended, since he takes his work as posing a challenge to the traditional approach to scientific rationality. In Lakatos's case, the fact that *anything goes* follows (quite unexpectedly) from the fact that his methodology of scientific research programs doesn't allow one to make rational choices among *rival* research programs (see [9]).

In this paper, I argue that it is possible to make sense of scientific rationality – even in the presence of inconsistencies – without being led to triviality. The claim goes through in either sense of triviality; that is, whether we take 'triviality' as paraconsistent logicians do (according to which every sentence in a given language is true) or in Feyerabend's sense (according to which, *anything goes*).

The main idea is to explore some consequences of the framework developed by da Costa and French in terms of partial structures and quasi-truth (see [5], [6], [7] and [8]). I argue that if the aim of science is understood as the search for quasi-true theories (rather than true ones), there is plenty of room for scientific rationality even in the presence of inconsistencies. Some examples are then presented to flesh out and illustrate how this account works.

2 The Problem of rationality

In order to model scientific rationality, two desiderata should be met. First, an account of rationality should be developed in such a way that it makes sense of the way in which we gather and assess evidence; in particular, it should make sense of how information is used and assessed in science. Secondly, judgments of rationality have a *normative* component; any account of rationality needs to address this feature as well.

What I have been calling here the *traditional approach* to rationality links rationality to the existence of substantial evidence to ground our beliefs. The crucial feature of the traditional

approach is nicely captured by what van Fraassen has called the 'Prussian concept of rationality' ([15], p. 171). According to the latter, what is rational to believe is precisely what we are *rationally compelled* to believe. Clearly, the Prussian concept introduces very stringent demands on rationality. As a result, a considerable amount of our beliefs turn out to be irrational under this construal. In particular, cases in which there are reasons for belief – although not *compelling* reasons – are simply considered to be examples of irrationality. Given that most of the evidence we obtain is at best partial, and given the limitations of our cognitive apparatus, only in very special cases do we have *compelling* reasons for belief. Thus, according to the Prussian account of rationality, most of our epistemic life is sheer irrationality.

This motivates the formulation of a more lenient account of rationality; one that van Fraassen has called 'the English concept of rationality' ([15], pp. 171-172). According to this account, what is rational to believe includes anything that we are *not* rationally compelled to *dis*believe. If the Prussian concept of rationality is too restrictive, the English concept, being the dual of the Prussian, is quite indulgent. We are entitled to rationally believe anything except those items for which we have rational grounds to disbelieve. The question then naturally arises: in attempting to avoid the extremes of the Prussian concept, has the English concept become *too* lenient?

The answer, of course, depends on what we take the things that are rationally compelling to believe (or disbelieve) to be. If we consider that we are only rationally compelled to disbelieve *P* if believing *P* engenders a contradiction, then take the account seems to be quite lenient. After all, only contradictions would be excluded from the process of rational belief. If we construe the notion of being rationally compelled to disbelieve as being constrained by probability assignments, then the English concept has some bite. It might not be rationally compelling to believe that there will be a really cold day in the middle of the summer in Fresno, given the small probability that this event will happen. In this sense, given that we might be rationally

compelled to disbelieve this claim, it would be irrational to believe it.

But maybe now we have overdone the point! Are we rationally compelled to disbelieve Bohr's atomic model? Are we rationally compelled to disbelieve quantum mechanics and general relativity (simultaneously)? Are we rationally compelled to disbelieve naïve set theory? What about Dirac's formulation of quantum mechanics (using the δ function)? Are we rationally compelled to disbelieve it? Should we also disbelieve the original formulation of the calculus?

Of course, what all these examples have in common is the fact that the above theories are inconsistent (or at least, as is the case of quantum mechanics and relativity theory, the theories are inconsistent with one another). It is also the case that these theories are important and have played (or still play) a crucial role in scientific activity. Even the English account of rationality makes the scientific community irrational for believing these theories. It then becomes clear that we need some alternative account of rationality that can make better sense of our epistemic situation.

In particular, any account of rationality needs to make sense of the fact that consistency is neither a necessary nor a sufficient condition for rationality. There are rational beliefs that are not consistent. For example, scientists accepted Bohr's model of the atom, although it wasn't consistent. On the other hand, there are consistent beliefs that are not rational. For example, it is perfectly consistent to believe that there is phlogiston; but is it *rational* to believe so?

Once this point is appreciated, it opens the way for an account of rationality that aims to be closer to our actual epistemic situation; an account in which instead of simply rejecting inconsistent theories as irrational, we can explore them and develop consistent alternatives based on them (see [7] and [8]). Of course, in order to articulate this proposal, one needs a new conceptual framework; a framework in which we can represent our epistemic situation and in which we can accommodate inconsistencies without triviality. To present the main features of this framework is the main point of the next section.

3 Partial structures and quasi-truth

The partial structures approach (as first presented in [12]) relies on three main notions: partial relation, partial structure and quasi-truth. One of the main motivations for introducing this proposal comes from the need for supplying a formal framework in which the 'openness' and 'incompleteness' of information dealt with in scientific practice can be accommodated in a unified way (see [5], [6], [7], and [8]). This is accomplished by extending, on the one hand, the usual notion of structure – in order to model the partialness of information we have about a certain domain (introducing then the notion of a *partial* structure) – and on the other hand, by generalizing the Tarskian characterization of the concept of truth for such 'partial' contexts (advancing the corresponding concept of *quasi-truth*).

In order to introduce a partial structure, the first step is to formulate an appropriate notion of *partial relation*. When investigating a certain domain of knowledge Δ, we formulate a conceptual framework that helps us in systematizing and organizing the information we obtain about Δ. This domain is tentatively represented by a set D of objects, and is studied by the examination of the relations holding among D's elements. The problem is that we often face the situation in which, given a certain relation R defined over D, we do not know whether all the objects of D (or n-tuples thereof) are related by R. This is part and parcel of the 'incompleteness' of our information about Δ, and is formally accommodated by the concept of partial relation. More formally, let D be a non-empty set; an n-place *partial relation* R over D is a triple $\langle R_1, R_2, R_3 \rangle$, where R_1, R_2, and R_3 are mutually disjoint sets, with $R_1 \cup R_2 \cup R_3 = D^n$, and such that: R_1 is the set of n-tuples that (we know that) belong to R, R_2 is the set of n-tuples that (we know that) do not belong to R, and R_3 is the set of n-tuples for which we do not know whether they belong or not to R. (Notice that if

R_3 is empty, R is a usual n-place relation which can be identified with R_1.)

However, in order to represent the information about the domain under consideration, we need a notion of *structure*. The following characterization, spelled out in terms of partial relations and based on the standard concept of structure, is meant to supply a notion that is broad enough to accommodate the partiality usually found in scientific practice. The partial relations do the main work, of course. A *partial structure S* is an ordered pair $\langle D, R_i \rangle_{i \in I}$, where D is a non-empty set, and $(R_i)_{i \in I}$ is a family of partial relations defined over D.

Two of the three basic notions of the partial structures approach are now defined. In order to spell out the last, and crucial one – quasi-truth – an auxiliary notion is required. The idea is to use, in the characterization of quasi-truth, the resources supplied by Tarski's definition of truth. However, since the latter is only defined for full structures, we have to introduce an intermediary notion of structure to 'link' full to partial structures. And this is the first role of those structures that extend a partial structure A into a full, total structure (which are called *A*-normal structures). Their second role is purely model-theoretic, namely to put forward an interpretation of a given language and, in terms of it, to characterise basic semantic notions. The question then is: how are *A*-normal structures to be defined? Here is an answer. Let $A = \langle D, R_i \rangle_{i \in I}$ be a partial structure. We say that the structure $B = \langle D', R'_i \rangle_{i \in I}$ is an *A-normal structure* if (i) $D = D'$, (ii) every constant of the language in question is interpreted by the same object both in A and in B, and (iii) R'_i extends the corresponding relation R_i (in the sense that each R'_i, supposed of arity n, is defined for all n-tuples of elements of D'). Notice that, although each R'_i is *defined* for all n-tuples over D', it holds for some of them (the R'_{i1}-component of R'_i), and it doesn't hold for others (the R'_{i2}-component).

As a result, given a partial structure A, there may be *too many A*-normal structures. Suppose that, for a given n-place partial relation R_i, we don't know whether $R_i a_1 ... a_n$ holds or not. One way of extending R_i into a full R'_i relation is to look for information to establish that it *does* hold, another way is to look for the contrary information. Both are *prima facie* possible ways of extending the partiality of R_i. But the same indeterminacy may be found with other objects of the domain, distinct from $a_1, ..., a_n$ (for instance, does $R_i b_1 ... b_n$ hold?), and with other relations distinct from R_i (for example, is $R_j b_1 ... b_n$ the case, with $j \neq i$?). In this sense, there are *too many* possible extensions of the partial relations that constitute A. Therefore we need to provide constraints to restrict the acceptable extensions of A.

In order to do that, we need first to formulate a further auxiliary notion (see [12]). A *pragmatic structure* is a partial structure to which a third component has been added: a set of accepted sentences P, which represents the accepted information about the structure's domain. (Depending on the interpretation of science which is adopted, different kinds of sentences are to be introduced in P: realists will typically include laws and theories, whereas empiricists will add mainly certain laws and observational statements about the domain in question.) A *pragmatic structure* is then a triple $A = \langle D, R_i, P \rangle_{i \in I}$, where D is a non-empty set, $(R_i)_{i \in I}$ is a family of partial relations defined over D, and P is a set of accepted sentences. The idea is that P introduces constraints on the ways that a partial structure can be extended.

The conditions for the existence of *A*-normal structures can now be spelled out (see [12]). Let $A = \langle D, R_i, P \rangle_{i \in I}$ be a pragmatic structure. For each partial relation R_i, we construct a set M_i of atomic sentences and negations of atomic sentences, such that the former correspond to the n-tuples that satisfy R_i, and the latter to those n-tuples that do not satisfy R_i. Let M be $\cup_{i \in I} M_i$. Therefore, a pragmatic structure A admits an *A*-normal structure if, and only if, the set $M \cup P$ is *consistent*.

Assuming that such conditions are met, we can now formulate the concept of quasi-truth. A sentence α is *quasi-true* in A according to B if (i) $A = \langle D, R_i, P \rangle_{i \in I}$ is a pragmatic structure, (ii) $B = \langle D', R'_i \rangle_{i \in I}$ is an *A*-normal structure, and (iii) α is true in B (in the Tarskian sense). If α is not quasi-true in A according to B, we say that α is

quasi-false (in *A* according to *B*). Moreover, we say that a sentence α is *quasi-true* if there is a pragmatic structure *A* and a corresponding *A*-normal structure *B* such that α is true in *B* (according to Tarski's account). Otherwise, α is *quasi-false*.

The idea, intuitively speaking, is that a quasi-true sentence α does not necessarily describe, in an appropriate way, the whole domain to which it refers, but only an aspect of it – the one modeled by the relevant partial structure *A*. After all, there are several different ways in which *A* can be extended to a full structure, and in some of these extensions α may not be true. As a result, the notion of quasi-truth is strictly weaker than truth: although every true sentence is (trivially) quasi-true, a quasi-true sentence is not necessarily true (since it may be false in certain extensions of *A*).

It may be argued that because quasi-truth has been defined in terms of full structures and the standard notion of truth, there is no gain with its introduction. In my view, there are several reasons why this is not the case. Firstly, as we have just seen, despite the use of full structures, quasi-truth is weaker than truth: a sentence which is quasi-true in a particular domain – that is, with respect to a given partial structure *A* – may not be true if considered in an extended domain. Thus, we have here a sort of 'underdetermination' – involving distinct ways of extending the same partial structure. This makes the notion of quasi-truth particularly suited to accommodate the underdetermination of theories by the data found in scientific practice.

Secondly, one of the points of introducing the notion of quasi-truth, as da Costa and French have argued in detail (see their [5], [6], [7]), is that in terms of this notion and the concept of partial structure, a formal framework can be advanced to accommodate the 'openness' and 'partialness' typically found in science and, in particular, in scientific practice. Bluntly put, the actual information at our disposal about a certain domain is modeled by a *partial* (but not full) structure *A*. Full, *A*-normal structures represent ways of extending the actual information which are possible according to *A*. In this respect, the use of full structures is a semantic expedient of the framework (in order to provide a definition of quasi-truth), but no epistemic import is assigned to them.

Thirdly, full structures can be ultimately dispensed with in the formulation of quasi-truth, since the latter can be characterized in a different way, in terms of quasi-satisfaction. This formulation preserves all the features of quasi-truth and is independent of the standard Tarskian type account of truth (see [1]). This provides, of course, the strongest argument for the dispensability of full structures (as well as of the Tarskian account) *vis-à-vis* quasi-truth. Therefore, full, *A*-normal structures are entirely inessential; their use here is only a convenient device.

To illustrate the use of quasi-truth, let us consider an example. As is well known, Newtonian mechanics is appropriate to explain the behavior of bodies under certain conditions (say, bodies which, roughly speaking, have 'low' velocity, are not subject to strong gravitational fields etc.). But with the formulation of special relativity, we know that if these conditions are not satisfied, Newtonian mechanics is false. In this sense, these conditions specify a family of partial relations, which delimit the context in which the theory holds. Although Newtonian mechanics is not true (and we know under what conditions it is false), it is *quasi-true*; that is, it is true in a given context, determined by a pragmatic structure and a corresponding *A*-normal one (see [7]).

4 Rationality, quasi-truth and inconsistency

The crucial feature of the present proposal is that if we understand the aim of science as being the search for quasi-true theories (rather than true ones), we can make perfect sense of how to keep rationality even in the presence of inconsistencies. As opposed to what happens with the Prussian and the English concepts of rationality, the present proposal doesn't exclude the possibility that we can rationally pursue inconsistent theories. After all, inconsistent theories can both be quasi-true. What we are rationally compelled to disbelieve are *trivial* theories.

And there is even an heuristic role in pursuing inconsistent theories: as Feyerabend correctly pointed out a long time ago, sometimes pursuing inconsistent theories is the only way of obtaining new information about the world (see [9]). The present framework can make perfect sense of this situation, by emphasizing that the quasi-truth of a given theory is relative to a given partial structure A. And to obtain new information about the original domain of investigation Δ, typically we need to explore different partial structures A'; in particular, we need to consider even partial structures that are *not consistent* with A (in the sense that there sentences that hold in A, but do not hold in A'). In doing this, triviality does not follow, of course, because not every theory is quasi-true.

We can now return to the two desiderata (discussed in section 2 above) that should be satisfied by any account of rationality. How does the present view accommodate them? The first desideratum insisted that an account of scientific rationality made sense of how information is used and assessed in science (hopefully in a rational way!). The present account is sensitive to the limitations of our cognitive situation and to the way in which we obtain information about the world. In fact, partial structures have been introduced to accommodate the facts that (i) typically we do not have complete information about a particular domain Δ; (ii) often there is more than one acceptable theory about Δ (scientific theories can be underdetermined by the data), and (iii) there are different ways of expanding the accepted partial information about Δ into a complete set of information (an A-normal structure). In this way, the first desideratum is satisfied, since we have here an account of rationality that is sensitive to crucial features of scientific practice and the process that we use to obtain information about the world.

The second desideratum highlighted the fact that rationality is a normative notion. The present view accommodates this requirement by insisting that not every theory is quasi-true. To believe in theories that are not quasi-true is to entertain an irrational belief. After all, if T is *not* quasi-true, then there is no extension of our current information in which T is true. In other words, to believe in a theory that is not quasi-true is to believe in a theory that can never be true.

It should now be clear how the present approach deals with rationality – even in the presence of inconsistencies – without triviality. Given that inconsistent theories can be both quasi-true, there is nothing irrational in pursuing them. In the case of Bohr's atomic model, for example, given the empirical success of the model and the lack of any consistent alternative at the time, it was perfectly sensible to pursue that model. Although not true, Bohr's model can be said to be quasi-true: it correctly describes a particular domain of quantum mechanics (the domain determined by the relevant partial structure A).

Naïve set theory also provides a fascinating example of how to rationally explore inconsistent conceptual alternatives without triviality. It's only in the context of such a theory (or at least in some paraconsistent version of an axiomatic set theory) that the properties of Russell set, for instance, can be explored (see [2], [3], and [14]). Again, even if naïve set theory is not true, at least it is quasi-true. It provides a description of a particular domain of the universe of sets, namely those sets that are inconsistent but non-trivial (in the sense that they have some properties but lack others).

Of course, in all these examples, it's crucial that the underlying logic be paraconsistent. Otherwise, the existence of inconsistencies would trivialize the whole system. This is a familiar point now: to make sense of inconsistencies without triviality some paraconsistent logic needs to be used. An important feature of the logic of quasi-true is that it is paraconsistent (see [4]). It's not surprising then that quasi-truth provides such a fruitful framework to deal with inconsistencies in science.

In conclusion, the present proposal offers an alternative way of approaching scientific rationality that is sensitive to scientific practice and is not trivialized by the presence of inconsistent theories. And although I am not suggesting that we should believe that inconsistent theories are true, we should

definitely explore them. After all, they may provide a *quasi-true* description of the world around us.

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