Paraconsistent Reasoning Based on Strong Relevant Logic [1]

(Extended Abstract)

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Abstract. *Both a body of human knowledge and a knowledge/information system may be incomplete and inconsistent in many ways. Reasoning with incomplete and inconsistent knowledge/information is the rule rather than the exception in our everyday real-life situations and most scientific disciplines. Paraconsistent reasoning, or reasoning in the presence of inconsistency, is indispensable to scientific discovery. This paper intends to answer such a fundamental question: What kind of logic system can satisfactorily underlie paraconsistent reasoning in scientific discovery? We show why classical mathematical logic, its various classical conservative extensions, and traditional (weak) relevant logics cannot satisfactorily underlie paraconsistent reasoning in scientific discovery, and propose that one should adopt strong relevant logic as the fundamental logic to underlie paraconsistent reasoning in scientific discovery.*

Key words: *Validity of reasoning, Conditional, Paraconsistent formal theory, Relevant logic, Strong relevance*

1. Introduction

Although reasoning and its automation was the most actively investigated subject in a long time in both Computer Science and Artificial Intelligence disciplines, there still many open problems concerning fundamental characteristics of reasoning and its effective and efficient implementation on computers.

Reasoning is the process of drawing new conclusions from some premises, which are known facts or assumed hypotheses. In general, a reasoning consists of a number of arguments (or inferences). An ***argument*** (or ***inference***) is a set of declarative sentences consisting of one or more sentences as its premises, which contain the evidence, and one sentence as its conclusion. In an argument, a claim is being made that there is some sort of ***evidential relation*** between its premises and its conclusion: the conclusion is supposed to ***follow from*** the premises, or equivalently, the premises are supposed to ***entail*** the conclusion. The correctness of an argument is a matter of the ***connection*** between its premises and its conclusion, and concerns the ***strength*** of the relation between them. Therefore, the correctness of an argument depends on the connection between its premises and its conclusion, and neither on whether the premises are true or not, nor on whether the conclusion is true or not. Thus, we have a fundamental question: What is the criterion by which one can decide whether the conclusion really does follow from the premises or not? A ***logically valid reasoning*** is a reasoning such that its arguments are justified based on some logical criterion in order to obtain correct conclusions. Today, there are so many different logic systems established based on different philosophical motivations. As a result, a reasoning may be valid on one logical criterion but invalid on another.

Generally, for any correct argument in scientific reasoning as well as our everyday reasoning, the conclusion of the argument must somehow be ***relevant*** to the premises of that argument, and vice versa. On the other hand, for any correct argument in a deductive reasoning, the conclusion of the argument must be true if all premises of that argument are true (in the sense of a certainly defined meaning of truth), i.e., any correct argument in a deductive reasoning must be ***truth-preserving***.

Proving is the process of finding a justification for a previously explicitly specified statement from some known facts or assumed hypotheses. A ***proof*** is a description of a found

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justification. A ***logically valid proving*** is a proving such that it is justified based on some logical criterion in order to obtain a correct proof.

Unfortunately, many studies in Computer Science and Artificial Intelligence disciplines confused and are still confusing reasoning and proving. Indeed, the most intrinsic difference between the notion of reasoning and the notion of proving is that the former is intrinsically prescriptive and predictive while the latter is intrinsically descriptive and non-predictive. The purpose of reasoning is to find some new conclusions previously unknown or unrecognized, while the purpose of proving is to find a justification for some statement previously known or assumed. Proving has an explicitly defined target as its goal while reasoning does not.

Since a reasoning has no previously explicitly defined target, the only criterion it must act according to is to reason correct conclusions when the premises are correct. It is logic that can underlie valid reasoning generally.

Both a body of human knowledge and a knowledge/information system may be incomplete and inconsistent in many ways. Reasoning with incomplete and inconsistent knowledge/information is the rule rather than the exception in our everyday real-life situations and most scientific disciplines. Paraconsistent reasoning, or reasoning in the presence of inconsistency, is indispensable to scientific discovery. Thus, we have a fundamental question: what kind of logic system can satisfactorily underlie paraconsistent reasoning in scientific discovery? This paper intends to answer the question. We show why classical mathematical logic, its various classical conservative extensions, and traditional (weak) relevant logics cannot satisfactorily underlie paraconsistent reasoning in scientific discovery, and propose that one should adopt strong relevant logic as the fundamental logic to underlie paraconsistent reasoning reasoning in scientific discovery.

2. The Notion of Conditional and Various Logic Systems

In various mathematical, natural, and social scientific literature, it is probably difficult, if not impossible, to find a sentence form that is more generally used for describing various definitions, propositions, theorems, and laws than the sentence form of 'if ... then ...'. In logic, a sentence of the form 'if ... then ...' is usually called a ***conditional proposition*** or simply ***conditional*** which states that there exists a relationship of sufficient condition between the 'if' part and the 'then' part of the sentence. Mathematical, natural, and social scientists always use conditionals in their descriptions of various definitions, propositions, theorems, and laws to connect a concept, fact, situation or conclusion and its sufficient conditions. Indeed, the major work of almost all scientists is to discover some sufficient condition relations between various phenomena, data, and laws in their research fields.

In general, a conditional must involve two parts which are connected by the connective 'if … then ... ' and called the ***antecedent*** and the ***consequent*** of that conditional, respectively. The truth-value of a conditional depends not only on the truth-values of its antecedent and consequent but also more essentially on a necessarily relevant and conditional relation between them. The notion of conditional plays the most essential role in reasoning because any reasoning form must invoke it, and therefore, it is historically always the most important subject studied in logic and is regarded as the heart of logic [1].

When we study and use logic, the notion of conditional may appear in both the object logic (i.e., the logic we are studying) and the meta-logic (i.e., the logic we are using to study the object logic). In the object logic, there usually is a connective in its formal language to represent the notion of conditional, and the notion of conditional is also usually used for representing a logical consequence relation in its proof theory or model theory. On the other hand, in the meta-logic, the notion of conditional, usually in the form of natural language, is used for defining various meta-notions and describe various meta-theorems about the object logic.

From the viewpoint of object logic, there are two classes of conditionals. One class is empirical conditionals and the other class is logical conditionals. For a logic, a conditional is called an ***empirical conditional*** of the logic if its truth-value, in the sense of that logic, depends on the contents of its antecedent and consequent and therefore cannot be determined only by its abstract form (i.e., from the viewpoint of that logic, the relevant relation between the antecedent and the consequent of that conditional is regarded to be empirical); a conditional is called a ***logical conditional*** of the logic if its truth-value, in the sense of that logic, depends only on its abstract form but not on the contents of its antecedent and consequent, and therefore, it is considered to be universally true or false (i.e., from the viewpoint of that logic, the relevant relation between the antecedent and the consequent of that conditional is regarded to be logical). A logical conditional that is considered to be universally true, in the sense of that logic, is also called an ***entailment*** of that logic. Indeed, the most intrinsic difference between various different logic systems is to regard what class of conditionals as entailments, as Diaz pointed out: "The problem in modern logic can best be put as follows: can we give an explanation of those conditionals that represent an entailment relation?" [6]

Logic deals with what entails what or what follows from what. Its aim is to determine the correct conclusions of a given set of premises, i.e., to determine that which arguments are ***valid***. Therefore, the most essential and central concept in logic is the ***logical consequence relation*** that relates a given set of premises to those conclusions, which validly follow from the premises. It is the difference between definitions (and formalizations) of the logical consequence relation leads to different logic systems, while it is the difference between philosophical considerations (and motivations) on logical validity criterion leads to different logical consequence relations.

In general, a ***formal logic system*** L consists of a formal language, denoted by F(L), which is the set of all ***well-formed formulas*** of L, and a ***logical consequence relation***, denoted by $\vdash_L$, such that for $P \subseteq F(L)$ and $t \in F(L)$, $P \vdash_L t$ means that within the framework of L, taking P as premises one can obtain t as a valid conclusion in the sense of L. For a formal logic system $(F(L), \vdash_L)$, a ***logical theorem*** t is a formula of L such that $\phi \vdash_L t$ where ϕ is the empty set. We use Th(L) to denote the set of all logical theorems of L. According to the representation of the logical consequence relation of a logic, the logic can be represented as a Hilbert style formal system, a Gentzen natural deduction system, a Gentzen sequent calculus system, or some other type of formal system.

Let $\neg$ be the negation certainly defined in a formal logic system L. L is said to be ***explosive*** if and only if $\{A, \neg A\} \vdash_L B$ for any two different formulas A and B; L is said to be ***paraconsistent*** if and only if it is not explosive [10]. Since the definition of paraconsistency is so simple, all logic systems can be simply divided into two classes: explosive logics and paraconsistent logics. Classical mathematical logic, its various classical conservative extensions, intuitionistic logic, and various modal logics are known to be explosive. However, obviously, the definition of paraconsistency concerns how to define the negation of a proposition. Also, how to interpret pair $\{A, \neg A\}$ semantically leads to various interesting problems.

3. Paraconsistent and Explosive Formal Theories

Let $(F(L), \vdash_L)$ be a formal logic system and $P \subseteq F(L)$ be a non-empty set of ***sentences*** (i.e., ***closed well-formed formulas***). A ***formal theory*** with premises P based on L, called an ***L-theory with premises P*** and denoted by $T_L(P)$, is defined as $T_L(P) =_{df} Th(L) \cup Th_L^e(P)$, and $Th_L^e(P) =_{df} \{et \mid P \vdash_L et \text{ and } et \notin Th(L)\}$ where Th(L) and $Th_L^e(P)$ are called the ***logical part*** and the ***empirical part*** of the formal theory, respectively, and any element of $Th_L^e(P)$ is called an ***empirical theorem*** of the formal theory. Fig.1 shows a formal theory.

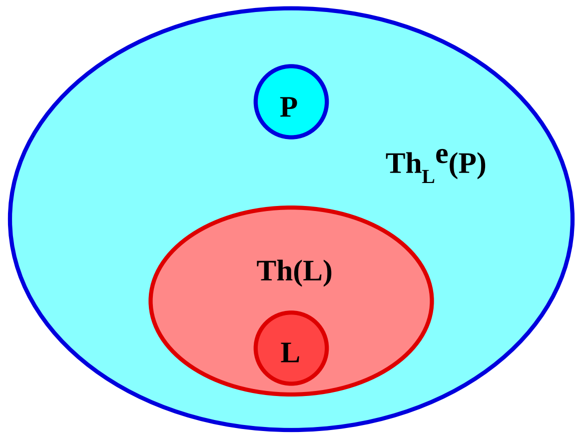

Fig. 1 Formal theory

Let $\neg$ be the negation certainly defined in a formal logic system L. A formal theory $T_L(P)$ is said to be ***directly inconsistent*** if and only if there exists a formula A of L such that both $A \in P$ and $\neg A \in P$ hold. A formal theory $T_L(P)$ is said to be ***indirectly inconsistent*** if and only if it is not directly inconsistent but there exists a formula A of L such that both $A \in T_L(P)$ and $\neg A \in T_L(P)$. A formal theory $T_L(P)$ is said to be ***consistent*** if and only if it is neither directly inconsistent nor indirectly inconsistent. In general, any formal theory may be indirectly inconsistent, without regard to that it is constructed as a purely deductive science (e.g., classical mathematical logic) or it is constructed based on some empirical or experimental science.

A formal theory $T_L(P)$ is said to be ***explosive*** if and only if $\{A, \neg A\} \vdash_L B$ for any two different formulas A and B; $T_L(P)$ is said to be ***paraconsistent*** if and only if it is not explosive. An explosive formal theory is not useful at all. Therefore, any meaningful formal theory constructed based on an empirical or experimental science should be paraconsistent. Note that if a formal logic system L is explosive, then any directly or indirectly inconsistent L-theory $T_L(P)$ must be explosive.

For a given formal theory $T_L(P)$ and any formula A of L, A is said to be ***explicitly accepted*** by $T_L(P)$, denoted by e-acc(A, $T_L(P)$), if and only if $A \in P$ and $\neg A \notin P$; A is said to be ***explicitly rejected*** by $T_L(P)$, denoted by e-rej(A, $T_L(P)$), if and only if $A \notin P$ and $\neg A \in P$; A is said to be ***explicitly inconsistent*** with $T_L(P)$, denoted by e-inc(A, $T_L(P)$), if and only if both $A \in P$ and $\neg A \in P$; A is said to be ***explicitly independent*** of $T_L(P)$ and is called an ***explicitly possible new premise*** for $T_L(P)$, denoted by e-ind(A, $T_L(P)$), if and only if both $A \notin P$ and $\neg A \notin P$. For any given formal theory $T_L(P)$ and any formula $A \notin P$, A is said to be ***implicitly accepted*** by $T_L(P)$, denoted by i-acc(A, $T_L(P)$), if and only if $A \in T_L(P)$ and $\neg A \notin T_L(P)$; A is said to be ***implicitly rejected*** by $T_L(P)$, denoted by i-rej(A, $T_L(P)$), if and only if $A \notin T_L(P)$ and $\neg A \in T_L(P)$; A is said to be ***implicitly inconsistent*** with $T_L(P)$, denoted by i-inc(A, $T_L(P)$), if and only if both $A \in T_L(P)$ and $\neg A \in T_L(P)$; A is said to be ***implicitly independent*** of $T_L(P)$ and is called an ***implicitly possible new premise*** for $T_L(P)$, denoted by i-ind(A, $T_L(P)$), if and only if both $A \notin T_L(P)$ and $\neg A \notin T_L(P)$.

According to the above definitions, an explicitly accepted formula may be either implicitly inconsistent or implicitly accepted; an explicitly rejected formula may be either implicitly inconsistent or implicitly rejected; an explicitly inconsistent formula must be implicitly inconsistent; an explicitly independent formula may be either implicitly inconsistent, or implicitly accepted, or implicitly rejected, or implicitly independent.

Table 1 shows that there are 9 different epistemic attitudes of any formula A of L for a formal theory $T_L(P)$.

4. On the Fundamental Logic to Underlie Paraconsistent Reasoning in Scientific Discovery

Until now, all the above discussions are general and not dependent on any special kind of logic system. Below we will discuss the role that various logic systems can play in scientific discovery.

The fundamental logic that can satisfactorily underlie scientific reasoning has to satisfy at least two important essential requirements. First, as a general logical criterion for validity of reasoning, the logic must be able to underlie relevant reasoning as well as truth-preserving reasoning in the sense of conditional. Second, the logic must be able to underlie ampliative, paracomplete, and paraconsistent reasoning; in particular, from the viewpoint of paraconsistency, the principle of Explosion that everything follows from a contradiction must not be accepted by the logic as a valid principle.

Classical mathematical logic (**CML** for short) was established in order to provide formal languages for describing the structures with which mathematicians work, and the methods of proof available to them. It is based on a number of fundamental assumptions. Some of the assumptions concerning our subject are as follows:

The ***classical abstraction***: The only properties of a proposition that matter to logic are its form and its truth-value.

The ***Fregean assumption***: The truth-value of a proposition is determined by its form and the truth-values of its constituents.

The ***classical account of validity***: An argument is valid if and only if it is impossible for all its premises to be true while its conclusion is false.

The ***Principle of Bivalence***: There are exactly two truth-values, TRUE and FALSE. Every declarative sentence has one or other, but not both, of these truth-values.

Obviously, the relevant relationship between the premises and conclusion of an argument is not accounted for by the classical validity criterion of **CML**. This results in that a reasoning based on **CML** is not necessarily relevant, i.e., its conclusion may be not relevant at all, in the sense of meaning and context, to its premises. In other words, in the framework of **CML**, even if a reasoning is valid in the sense of the classical account of validity, the relevance relationship between its premises and its conclusion cannot be guaranteed necessarily. This approach, however, may be suitable to searching and describing a formal proof of a previously specified theorem, but not necessarily suitable to forming a new concept and discovering a new theorem because the aim, nature, and role of the **CML** is descriptive and non-predictive rather than prescriptive and predictive.

The explosiveness of **CML** is also a result of the classical validity criterion and the principle of bivalence. The principle of Explosion ("ex falso quodlibet"), i.e., the inference from A and $\neg A$ to B, where A and B are any two formulas, is valid in **CML**. **CML** has a logical theorem $(A \wedge \neg A) \rightarrow B$ corresponding to "ex falso quodlibet". As a result, in the framework of **CML**, reasoning under inconsistency is impossible because any formal theory $T_{\mathbf{CML}}(P)$ must be explosive if it is directly or indirectly inconsistent. However, as we have pointed out, almost all formal theories based on empirical or experimental sciences generally may be indirectly inconsistent. This problem also exists in any classical conservative extension or non-classical alternative of **CML** where $(A \wedge \neg A) \rightarrow B$ is accepted as a logical theorem and Modus Ponens for material implication serves as an inference rule.

Relevant logics were constructed during the 1950s~1970s in order to find a satisfactory way of grasping the notion of conditional and avoiding the so-called "implicational paradoxes" [1, 2, 7, 11]. Some major traditional relevant logic systems are "system **E** of entailment", "system **R** of relevant implication", and "system **T** of ticket entailment". A major feature of the relevant logics is that they have a primitive intensional connective to represent the notion of conditional and their logical theorems include no implicational paradoxes. The underlying principle of the relevant logics is the ***relevance principle***, i.e., for any entailment provable in **T**, **E**, or **R**, its antecedent and consequent must share a sentential variable. ***Variable-sharing*** is a formal notion designed to reflect the idea that there be a meaning-connection between the antecedent and consequent of an entailment [1, 2, 7, 11]. It is this relevance principle that rejects the classical account of validity and excludes those implicational paradoxes from logical axioms or theorems of relevant logics. As a typical implicational paradox, $(A\wedge\neg A)\rightarrow B$ is rejected by any relevant logic because there is no variable shared by the antecedent and consequent. Therefore, the major traditional relevant logic systems are paraconsistent.

However, although the traditional relevant logics have rejected those implicational paradoxes, there still exist some logical axioms or theorems in the logics, which are not natural in the sense of conditional. Such logical axioms or theorems, for instance, are $(A\wedge B)\Rightarrow A$, $(A\wedge B)\Rightarrow B$, $(A\Rightarrow B)\Rightarrow((A\wedge C)\Rightarrow B)$, $A\Rightarrow(A\vee B)$, $B\Rightarrow(A\vee B)$, $(A\Rightarrow B)\Rightarrow(A\Rightarrow(B\vee C))$ and so on, where $\Rightarrow$ denotes the primitive intensional connective in the logics to represent the notion of conditional. The present author named these logical axioms or theorems "***conjunction-implicational paradoxes***" and "***disjunction-implicational paradoxes***" [3-5]. The present author thinks that it is problematical to regard these conditionals as entailments.

Let us discuss the problem from the viewpoint of paraconsistency. Since $(A\wedge B)\Rightarrow A$ and $(A\wedge B)\Rightarrow B$ are logical theorems of all traditional relevant logics, $(A\wedge\neg A)\Rightarrow A$ and $(A\wedge\neg A)\Rightarrow\neg A$ are valid in the relevant logics. The definition of paraconsistent logic is restrictive but not definitive, i.e., it just says that any explosive logic is not a paraconsistent logic, but not so definitively says what a paraconsistent logic should be. Now, we have a question, if $(A\wedge\neg A)\Rightarrow A$ and $(A\wedge\neg A)\Rightarrow\neg A$ should be accepted as entailments by any paraconsistent logic, then what philosophical interpretations can be given to them?

Moreover, for any formal theory $T_{\mathbf{T}}(P)$, $T_{\mathbf{E}}(P)$, or $T_{\mathbf{R}}(P)$, all conjunction-implicational and disjunction-implicational paradoxes are logical theorems of $T_{\mathbf{T}}(P)$, $T_{\mathbf{E}}(P)$, or $T_{\mathbf{R}}(P)$. As a result, from any given premise $A\Rightarrow B$, we can infer $(A\wedge\neg A)\Rightarrow B$, $(A\wedge\neg A\wedge D)\Rightarrow B$, and so on by using the logical theorem $(A\Rightarrow B)\Rightarrow((A\wedge C)\Rightarrow B)$ of **T**, **E**, and **R** and Modus Ponens for conditional, i.e., $(A\wedge\neg A)\Rightarrow B\in T_{\mathbf{T/E/R}}(P)$, $(A\wedge\neg A\wedge D)\Rightarrow B\in T_{\mathbf{T/E/R}}(P)$, ... for any $A\Rightarrow B\in T_{\mathbf{T/E/R}}(P)$. That is, although $(A\wedge\neg A)\Rightarrow B$ is not a logical theorem of any major traditional relevant logic, it can certainly be an empirical theorem of any formal theory with premises P, which includes $A\Rightarrow B$, based on any major traditional relevant logic. How should we consider this situation? Is it rational to our everyday real-life situations and scientific disciplines? The present author's answer is "NO."

Note that major paraconsistent logics C_n ($1 \leq n \leq \omega$) proposed by da Costa are not relevant. Implicational paradox $A\rightarrow(B\rightarrow A)$, conjunction-implicational paradoxes $(A\wedge B)\Rightarrow A$ and $(A\wedge B)\Rightarrow B$, and disjunction-implicational paradoxes $A\Rightarrow(A\vee B)$ and $B\Rightarrow(A\vee B)$ are logical axioms of them [10].

In order to establish a satisfactory logic calculus of conditional to underlie relevant reasoning, the present author has proposed some ***strong relevant logics***, named **Rc**, **Ec**, and **Tc**, and shown their applications in knowledge engineering [3-5]. As a modification of traditional relevant logics **R**, **E**, and **T**, strong relevant logics **Rc**, **Ec**, and **Tc** reject all conjunction-implicational paradoxes and disjunction-implicational paradoxes in **R**, **E**, and **T**, respectively. Therefore they are free not only of implicational paradoxes but also of conjunction-implicational and disjunction-implicational paradoxes. What underlies the strong relevant logics is the ***strong relevance principle***. We say that a logic system satisfies the ***strong relevance principle*** if for any logical theorem of the logic, say A, every sentential variable in A occurs at least once as an antecedent part and at least once as a consequent part. Those traditional relevant logics that only satisfy the relevance principle but not the strong relevance principle can be called the ***weak relevant logics***.

From the viewpoint that the fundamental logic to underlie scientific discovery should satisfactorily underlie relevant, ampliative, paracomplete, and paraconsistent reasoning, strong relevant logics are hopeful candidates better than classical mathematical logic, its various classical conservative extensions, traditional (weak) relevant logics, and traditional paraconsistent logics.

5. Paraconsistent Reasoning Based on Strong Relevant Logic in Scientific Discovery

Perhaps the best known work on modeling epistemic processes is the so-called AGM theory or AGM model [8, 9, 12]. In the AGM model, an epistemic state or belief set of an agent is represented by a deductively closed and consistent set of sentences in the propositional language of **CML**, denoted by Cn(K) = K where K is a set of sentences and Cn is the logical consequence relation of **CML**. For a belief set K and a sentence A, there are three basic operations in the model, i.e., expansion of K by A, contraction of K by A, and revision of K by A, where only two operations are independent. The model is thus defined by three groups of rationality postulates for the three basic operations, respectively, which are intended to grasp the notion of belief changes. In addition to these postulates, there is an ordering defined on the beliefs, called the epistemic entrenchment, which is used to ensure that there exists a unique belief change satisfying the constraints imposed by the postulates.

The AGM model adopts **CML**, which is explosive and has a great number of implicational paradoxes, as the fundamental logic to underlie epistemic processes. As a result, it does not allow inconsistent belief sets and cannot assure us of the validity of a belief in the form of conditional in any epistemic state even if all premises in the primary epistemic state are true or valid. Our strong relevant logic model of epistemic processes adopts predicate strong relevant logic **EcQ** as the fundamental logic to underlie epistemic processes. **EcQ** is paraconsistent and is free not only of implicational paradoxes but also of conjunction-implicational and disjunction-implicational paradoxes, and therefore, it allows for inconsistent belief sets and assures us of the validity of a belief in the form of conditional in any epistemic state, if all premises in the primary epistemic state are true or valid.

The AGM model keeps a belief set consistent at any time. However, the postulate that every belief set of an agent is consistent is too ideal, and therefore, is not natural and rational. To find out some inconsistency in an existent theory and then to reconstruct the theory such that the inconsistency will not exist in the new theory is often an epistemic process of a scientist in scientific discovery. To simply assume that every belief set of an agent is consistent must result in the neglect of investigating how to reason under inconsistency that is an ordinary work of many scientists in scientific discovery. Indeed, it is impossible to investigate the issue of reasoning under inconsistency within the framework of **CML**. Since **EcQ** is paraconsistent, our strong relevant logic model does not require belief sets must be consistent at any time.

6. Concluding Remarks

The ultimate goal of our research is to provide scientists with a computational methodology and some computational tools to program their epistemic processes in scientific discovery, and therefore, to make scientific discovery become a 'science' and/or an 'engineering'. To this end, based on the strong relevant logic model of epistemic processes, we have proposed a novel programming paradigm, named '***Epistemic Programming***' which is different from the existing programming paradigms in that it regards conditionals as the subject of computing, takes primary epistemic operations as basic operations of computing, and regards epistemic processes as the subject of programming [5].

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Table 1. Possible epistemic attitudes

	P	$T_L(P)$	P	$T_L(P)$	P	$T_L(P)$	P	$T_L(P)$
A	$\in$	$\in$	$\in$	$\in$	$\in$	$\in$	$\in$	$\in$
$\neg$A	$\in$	$\in$	$\in$	$\notin$	$\notin$	$\in$	$\notin$	$\notin$
Attitudes	e-inc	i-inc	e-inc	i-acc	e-acc	i-inc	e-acc	i-acc
Attitudes	(1) e-inc-i-inc		impossible		(2) e-acc-i-inc		(3) e-acc-i-acc	

	P	$T_L(P)$	P	$T_L(P)$	P	$T_L(P)$	P	$T_L(P)$
A	$\in$	$\notin$	$\in$	$\notin$	$\in$	$\notin$	$\in$	$\notin$
$\neg$A	$\in$	$\in$	$\in$	$\notin$	$\notin$	$\in$	$\notin$	$\notin$
Attitudes	e-inc	i-rej	e-inc	i-ind	e-acc	i-rej	e-acc	i-ind
Attitudes	impossible		impossible		impossible		impossible	

	P	$T_L(P)$	P	$T_L(P)$	P	$T_L(P)$	P	$T_L(P)$
A	$\notin$	$\in$	$\notin$	$\in$	$\notin$	$\in$	$\notin$	$\in$
$\neg$A	$\in$	$\in$	$\in$	$\notin$	$\notin$	$\in$	$\notin$	$\notin$
Attitudes	e-rej	i-inc	e-rej	i-acc	e-ind	i-inc	e-ind	i-acc
Attitudes	(4) e-rej-i-inc		impossible		(5) e-ind-i-inc		(6) e-ind-i-acc	

	P	$T_L(P)$	P	$T_L(P)$	P	$T_L(P)$	P	$T_L(P)$
A	$\notin$	$\notin$	$\notin$	$\notin$	$\notin$	$\notin$	$\notin$	$\notin$
$\neg$A	$\in$	$\in$	$\in$	$\notin$	$\notin$	$\in$	$\notin$	$\notin$
Attitudes	e-rej	i-rej	e-rej	i-ind	e-ind	i-rej	e-ind	i-ind
Attitudes	(7) e-rej-i-rej		impossible		(8) e-ind-i-rej		(9) e-ind-i-ind	