Strong Relevance as a Logical Validity Criterion for Scientific Reasoning [1]

(Extended Abstract)

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Abstract. *The purpose of any scientific reasoning in any field is to find some both new and interesting facts, concepts, and principles from known facts or assumed hypotheses. The logical validity of reasoning is the only criterion that any scientific reasoning must act according to in order to obtain correct conclusions from the premises of the reasoning. It is logic that can underlie valid reasoning. Until now, many studies on fundamental characteristics of scientific reasoning are still based on classical mathematical logic or its various classical conservative extensions. However, the classical validity criterion underlying the classical mathematical logic is not adequate to accomplishing the purpose of a scientific reasoning. This paper proposes that one should adopt the strong relevance between conclusions and premises as a logical validity criterion for any scientific reasoning in order to accomplish the purpose of scientific reasoning.*

Keywords*: Validity of reasoning, Conditional, Entailment, Relevant logic, The principle of strong relevance*

1. Introduction

The purpose of any scientific reasoning in any field is to find some both new and interesting facts, concepts, and principles from known facts or assumed hypotheses. The logical validity of reasoning is the only criterion that any scientific reasoning must act according to in order to obtain correct conclusions from the premises of the reasoning.

Reasoning is the process of drawing new conclusions from some premises, which are known facts or assumed hypotheses. In general, a reasoning consists of a number of arguments (or inferences). An ***argument*** (or ***inference***) is a set of declarative sentences consisting of one or more sentences as its premises, which contain the evidence, and one sentence as its conclusion. In an argument, a claim is being made that there is some sort of ***evidential relation*** between its premises and its conclusion: the conclusion is supposed to ***follow from*** the premises, or equivalently, the premises are supposed to ***entail*** the conclusion. The correctness of an argument is a matter of the ***connection*** between its premises and its conclusion, and concerns the ***strength*** of the relation between them. Therefore, the correctness of an argument depends on the connection between its premises and its conclusion, and neither on whether the premises are true or not, nor on whether the conclusion is true or not. Thus, we have a fundamental question: What is the criterion by which one can decide whether the conclusion really does follow from the premises or not? A ***logically valid reasoning*** is a reasoning such that its arguments are justified based on some logical criterion in order to obtain correct conclusions. Today, there are so many different logic systems established based on different philosophical motivations. As a result, a reasoning may be valid on one logical criterion but invalid on another.

Generally, for any correct argument in scientific reasoning as well as our everyday reasoning, the conclusion of the argument must somehow be ***relevant*** to the premises of that argument, and vice versa. On the other hand, for any correct argument in a deductive reasoning, the conclusion of the argument must be true if all premises of that argument are true (in the sense of a certainly defined meaning of truth), i.e., any correct argument in a deductive reasoning must be ***truth-preserving***.

Proving is the process of finding a justification for a previously explicitly specified

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statement from some known facts or assumed hypotheses. A ***proof*** is a description of a found justification. A ***logically valid proving*** is a proving such that it is justified based on some logical criterion in order to obtain a correct proof.

Unfortunately, many studies in Computer Science and Artificial Intelligence disciplines confused and are still confusing reasoning and proving. Indeed, the most intrinsic difference between the notion of reasoning and the notion of proving is that the former is intrinsically prescriptive and predictive while the latter is intrinsically descriptive and non-predictive. The purpose of reasoning is to find some new conclusions previously unknown or unrecognized, while the purpose of proving is to find a justification for some statement previously known or assumed. Proving has an explicitly defined target as its goal while reasoning does not.

Since a reasoning has no previously explicitly defined target, the only criterion it must act according to is to reason correct conclusions when the premises are correct. It is logic that can underlie valid reasoning generally.

Until now, many studies on fundamental characteristics of scientific reasoning are still based on classical mathematical logic (**CML** for short) or its various classical conservative extensions. However, the classical validity criterion underlying the **CML** is not adequate to accomplishing the purpose of a scientific reasoning. This paper proposes that one should adopt the strong relevance between conclusions and premises as a logical validity criterion for any scientific reasoning in order to accomplish the purpose of scientific reasoning.

2. The Notion of Conditional in Scientific Reasoning

In various mathematical, natural, and social scientific literature, it is probably difficult, if not impossible, to find a sentence form that is more generally used for describing various definitions, propositions, theorems, and laws than the sentence form of 'if ... then ...'. In logic, a sentence of the form 'if ... then ...' is usually called a ***conditional proposition*** or simply ***conditional*** which states that there exists a relationship of sufficient condition between the 'if' part and the 'then' part of the sentence. Mathematical, natural, and social scientists always use conditionals in their descriptions of various definitions, propositions, theorems, and laws to connect a concept, fact, situation or conclusion and its sufficient conditions. Indeed, the major work of almost all scientists is to discover some sufficient condition relations between various phenomena, data, and laws in their research fields.

In general, a conditional must involve two parts which are connected by the connective 'if … then ... ' and called the ***antecedent*** and the ***consequent*** of that conditional, respectively. The truth-value of a conditional depends not only on the truth-values of its antecedent and consequent but also more essentially on a necessarily relevant and conditional relation between them. The notion of conditional plays the most essential role in reasoning because any reasoning form must invoke it, and therefore, it is historically always the most important subject studied in logic and is regarded as the heart of logic [1].

When we study and use logic, the notion of conditional may appear in both the object logic (i.e., the logic we are studying) and the meta-logic (i.e., the logic we are using to study the object logic). In the object logic, there usually is a connective in its formal language to represent the notion of conditional, and the notion of conditional is also usually used for representing a logical consequence relation in its proof theory or model theory. On the other hand, in the meta-logic, the notion of conditional, usually in the form of natural language, is used for defining various meta-notions and describe various meta-theorems about the object logic.

From the viewpoint of object logic, there are two classes of conditionals. One class is empirical conditionals and the other class is logical conditionals. For a logic, a conditional is called an ***empirical conditional*** of the logic if its truth-value, in the sense of that logic, depends on the contents of its antecedent and consequent and therefore cannot be determined only by its abstract form (i.e., from the viewpoint of that logic, the relevant relation between the antecedent and the consequent of that conditional is regarded to be empirical); a conditional is called a ***logical conditional*** of the logic if its truth-value, in the sense of that logic, depends only on its abstract form but not on the contents of its antecedent and consequent, and therefore, it is considered to be universally true or false (i.e., from the viewpoint of that logic, the relevant relation between the antecedent and the consequent of that conditional is regarded to be logical). A logical conditional that is considered to be universally true, in the sense of that logic, is also called an ***entailment*** of that logic. Indeed, the most intrinsic difference between various different logic systems is to regard what class of conditionals as entailments, as Diaz pointed out: "The problem in modern logic can best be put as follows: can we give an explanation of those conditionals that represent an entailment relation?" [8]

Recently, the present author proposed some fundamental observations and assumptions on scientific discovery processes and their automation as follows [7]:

(1) **New conditionals are epistemic goals of any scientific discovery**: Any scientific discovery process must include an epistemic process to gain knowledge of or to ascertain the existence of some

empirical or logical conditionals previously unknown or unrecognized. Finding some new data or some new fact is just an initial step in a scientific discovery but not the scientific discovery itself.

(2) **Specific knowledge is the power of a scientist**: Any scientist who made a scientific discovery must have worked in some particular scientific field and more specifically on some problem in a particular domain within the field. There is no universal scientist who can make scientific discoveries in every field.

(3) **Any scientific discovery has an ordered epistemic process**: Any scientific discovery must have, among other things, a process that consists of a number of ordered epistemic activities that may be contributed by many scientists in a long duration. Any scientific discovery is neither an event occurring in a moment nor an accumulation of disorderly and disorganized inquiries.

(4) **Scientific reasoning is indispensable to any scientific discovery**: Any discovery must be previously unknown or unrecognized before the completion of discovery process. Reasoning is the only way to draw new conclusions from some premises that are known facts or assumed hypotheses. There is no scientific discovery that does not invoke scientific reasoning.

(5) **Scientific reasoning must be justified based on some sound logical criterion**: The most intrinsic difference between discovery and proof is that discovery has no explicitly defined target as its goal. Since any epistemic process in any scientific discovery has no explicitly defined target, the only criterion the epistemic process must act according to is to reason correct conclusions from the premises. It is logic that can underlie valid scientific reasoning.

(6) **Scientific reasoning must be truth-preserving**: For any argument to be correct in scientific reasoning as well as our everyday reasoning, the conclusion of the argument must be true if all premises of that argument are true. The meaning of truth must be in the sense of conditional as well as fact.

(7) **Scientific reasoning must be relevant**: For any argument to be correct in scientific reasoning as well as our everyday reasoning, the premises of the argument must be in some way relevant to the conclusion of that argument, and vice versa. A reasoning including some irrelevant arguments cannot be said to be valid in general.

(8) **Scientific reasoning must be ampliative**: A scientific reasoning is intrinsically different from a scientific proving in that the purpose of reasoning is to find out some facts and conditionals previously unknown or unrecognized, while the purpose of proving is to find out a justification for some fact previously known or assumed. A reasoning in any scientific discovery must be ampliative such that it enlarges or increases the reasoning agent's knowledge in some way.

(9) **Scientific reasoning must be paracomplete**: Any scientific theory may be incomplete in many ways, i.e., for some sentence 'A' neither it nor its negation can be true in the theory. Therefore, a reasoning in any scientific discovery must be paracomplete such that it does not necessarily reason out a sentence even if it cannot reason out the negation of that sentence.

(10) **Scientific reasoning must be paraconsistent**: Any scientific theory may be inconsistent in many ways, i.e., it may directly or indirectly include some contradiction such that for some sentence 'A' both it and its negation can be true together in the theory. Therefore, a reasoning in any scientific discovery must be paraconsistent such that from a contradiction it does not necessarily reason out an arbitrary sentence.

(11) **Epistemic activities in any scientific discovery process are identifiable and distinguishable**: Epistemic activities in any scientific discovery process can be identified and distinguished from other activities, e.g., experimental activities, as explicitly described thoughts.

(12) **Normal scientific discovery processes are possible**: Any scientific discovery process can be described and modeled in a normal way, and therefore, it can be simulated by computer programs automatically.

(13) **Specific knowledge is the power of a program**: Even if scientific discovery processes can be simulated by computer programs automatically in general, a particular computational process which can adequately perform a particular scientific discovery must take sufficient knowledge specific to the subject under investigation into account. There is no generally organized order of scientific discovery processes that can be applied to every problem in every field.

(14) **Any automated scientific discovery process must be valid**: Any automated process of scientific discovery has to assure us of the truth, in the sense of not only fact but also conditional, of the final result produced by the process if it starts from an epistemic state where all facts, hypotheses, and conditionals are regarded to be true or valid.

(15) **Any automated scientific discovery process need an autonomous forward reasoning mechanism**: Any backward or refutation deduction system cannot serve as an autonomous reasoning mechanism to form or discover some completely new things. What we need in automating scientific discovery is an autonomous forward reasoning mechanism.

According to the above fundamental observations and assumptions on scientific discovery processes and their automation, to establish a satisfactory criterion to evaluate whether or not the conclusion of a scientific reasoning is correct in the sense of conditional is the most crucial issue in studies of fundamental characteristics of scientific reasoning. If we do not have such a criterion, how can we evaluate and then decide whether the conclusion of a scientific reasoning is correct or not?

3. The Classical Validity Criterion

The **CML** was established in order to provide formal languages for describing the structures with which mathematicians work, and the methods of proof available to them. It is based on a number of fundamental assumptions. Some of the assumptions concerning our subject are as follows:

The ***classical abstraction***: The only properties of a proposition that matter to logic are its form and its truth-value.

The ***Fregean assumption***: The truth-value of a proposition is determined by its form and the truth-values of its constituents.

The ***classical account of validity***: An argument is valid if and only if it is impossible for all its premises to be true while its conclusion is false.

The ***Principle of Bivalence***: There are exactly two truth-values, T and F. Every declarative sentence has one or other, but not both, of these truth-values.

Obviously, the relevant relationship between the premises and conclusion of an argument is not accounted for by the classical validity criterion of **CML**. As a result, for a reasoning based on **CML** or its various classical conservative extensions, its conclusion may be not relevant at all, in the sense of meaning and context, to its premises. In fact, in the framework of **CML**, even if a reasoning is valid in the sense of classical account of validity, the relevance between its premises and its conclusion cannot be guaranteed necessarily. This approach, however, may be suitable to searching and describing a formal proof for a previously specified theorem, but not necessarily suitable to forming a new concept and discovering a new theorem because the aim, nature, and role of the **CML** is descriptive and non-predictive rather than prescriptive and predictive.

On the other hand, taking the above assumptions into account, in **CML**, the notion of conditional, which is intrinsically intensional but not truth-functional, is represented by the truth-functional extensional notion of ***material implication*** (denoted by $\rightarrow$ in this paper) that is defined as $A\rightarrow B =_{df} \neg(A\wedge\neg B)$ or $A\rightarrow B =_{df} \neg A\vee B$. This definition of material implication, with the inference rule of Modus Ponens for material implication (from A and $A\rightarrow B$ to infer B), can adequately satisfy the ***truth-preserving*** requirement of **CML,** i.e., the conclusion of a valid reasoning based on **CML** must be true (in the sense of **CML**) if all premises of the reasoning are true (in the sense of **CML**). This requirement is basic and adequate for **CML** to be used as a formal description tool by mathematicians.

However, the material implication is intrinsically different from the notion of conditional in meaning (semantics). It is no more than an extensional truth-function of its antecedent and consequent but does not require that there is a necessarily relevant and conditional relation between its antecedent and consequent, i.e., the truth-value of the formula $A\rightarrow B$ depends only on the truth-values of A and B, though there could exist no necessarily relevant and conditional relation between A and B. It is this intrinsic difference in meaning between the notion of material implication and the notion of conditional that leads to the well-known 'implicational paradox problem' in **CML**. The problem is that if one regards the material implication as the notion of conditional and regards every logical theorem of **CML** as a valid reasoning form or entailment, then a great number of logical axioms and logical theorems of **CML**, such as $A\rightarrow(B\rightarrow A)$, $B\rightarrow(\neg A\vee A)$, $\neg A\rightarrow(A\rightarrow B)$, $(\neg A\wedge A)\rightarrow B$, $(A\rightarrow B)\vee(\neg A\rightarrow B)$, $(A\rightarrow B)\vee(A\rightarrow\neg B)$, $(A\rightarrow B)\vee(B\rightarrow A)$, $((A\wedge B)\rightarrow C)\rightarrow((A\rightarrow C)\vee(B\rightarrow C))$, and so on, present some paradoxical properties and therefore they have been referred to in the literature as "***implicational paradoxes***" [1, 2, 9, 10]. "$B\rightarrow(\neg A\vee A)$" and "$(\neg A\wedge A)\rightarrow B$" are two most notorious paradoxes of material implication because they are regarded as to be logically true in the sense of **CML** but there is no relevant relationship at all between their antecedents and consequents.

Because all implicational paradoxes are logical theorems of any **CML**-theory $T_{\mathbf{CML}}(P)$, for a conclusion of a reasoning from a set P of premises based on **CML**, we cannot directly accept it as a correct conclusion in the sense of conditional, even if each of the given premises is regarded to be true and the conclusion can be regarded to be true in the sense of material implication. For example, from any given premise A, we can infer $B\rightarrow A$, $C\rightarrow A$, … where B, C, ... are arbitrary formulas, by using the logical axiom $A\rightarrow(B\rightarrow A)$ of **CML** and Modus Ponens for material implication, i.e., $B\rightarrow A\in T_{\mathbf{CML}}(P)$, $C\rightarrow A\in T_{\mathbf{CML}}(P)$, ... for any $A\in T_{\mathbf{CML}}(P)$. However, from the viewpoint of scientific reasoning as well as our everyday reasoning, these inferences cannot be regarded to be valid in the sense of conditional because there may be no necessarily relevant and conditional relation between B, C, ... and A and therefore we cannot say "if B then A", "if C then A", and so on. Obviously, no scientists did or will reason in such a way in their scientific discovery. This situation means that from the viewpoint of conditional or entailment, the truth-preserving property of reasoning based on **CML** is meaningless.

Consequently, in the framework of **CML**, even if a reasoning is valid, neither the truth of its conclusion in the sense of conditional nor the necessary relevance between its premises and its conclusion can be guaranteed necessarily. This is a direct result of the classical account of validity.

Note that all of various classical conservative extensions of **CML** where the notion of conditional is directly or indirectly represented by the material implication have the similar problems as the above problems in **CML**. For example, the main aim of Lewis's work beginning in 1912 on the establishment of modern modal logic was to find a satisfactory theory of implication which is better than **CML** in that it can avoid those implicational paradoxes. A major reason to say that Lewis's plan was not completely successful is that the two most notorious paradoxes of material implication "$B\rightarrow(\neg A\vee A)$" and "$(\neg A\wedge A)\rightarrow B$" are still remained in various modal logics as logical theorems in the term of strict implication.

4. The Relevant Validity Criterion

Generally, for any correct argument in scientific reasoning as well as our everyday reasoning, the premises of the argument must somehow be relevant to the conclusion of that argument, and vice versa. Informally, we may define that a reasoning is ***relevant*** if and only if in every argument or inference of that reasoning, the premises are relevant to the conclusion in the sense of meaning.

As we have pointed out, a reasoning based on **CML** and its various classical conservative extensions is not necessarily relevant even though it is truth-preserving (again, in the sense of **CML**).

Relevant logics were constructed during the 1950s~1970s in order to find a satisfactory way of grasping the notion of conditional and avoiding the paradoxes of material (and strict) implication [1, 2, 9, 10]. Some major traditional relevant logic systems are "system **E** of entailment", "system **R** of relevant implication", and "system **T** of ticket entailment". A major feature of the relevant logics is that they have a primitive intensional connective to represent the notion of conditional and their logical theorems include no implicational paradoxes. The underlying principle of the relevant logics is the ***relevance principle***, i.e., for any entailment provable in **T**, **E**, or **R**, its antecedent and consequent must share a sentential variable. ***Variable-sharing*** is a formal notion designed to reflect the idea that there be a meaning-connection between the antecedent and consequent of an entailment [1, 2, 9, 10]. It is this relevance principle that rejects the classical account of validity and excludes those implicational paradoxes from logical axioms or theorems of relevant logics.

However, although the traditional relevant logics have rejected those implicational paradoxes, there still exist some logical axioms or theorems in the logics, which are not natural in the sense of conditional. Such logical axioms or theorems, for instance, are $(A\wedge B)\Rightarrow A$, $(A\wedge B)\Rightarrow B$, $(A\Rightarrow B)\Rightarrow((A\wedge C)\Rightarrow B)$, $A\Rightarrow(A\vee B)$, $B\Rightarrow(A\vee B)$, $(A\Rightarrow B)\Rightarrow(A\Rightarrow(B\vee C))$ and so on, where $\Rightarrow$ denotes the primitive intensional connective in the logics to represent the notion of conditional. The present author named these logical axioms or theorems "***conjunction-implicational paradoxes***" and "***disjunction-implicational paradoxes***" [3, 5]. The reason to regard these logical axioms or theorems of the traditional relevant logics as paradoxes is that in the antecedent of a conjunction-implicational paradox there is a conjunct irrelevant to its consequent, and in the consequent of a disjunction-implicational paradox there is a disjunct irrelevant to its antecedent. This reason is similar to the case to regard those logical axioms or theorems of **CML**, whose antecedent are irrelevant to their consequent, as paradoxes. In other words, if we cannot regard a conditional whose antecedent and consequent shares no variable as an entailment, why we regard a conditional where some variable does not shared by its antecedent and consequent as an entailment without resistance? It seems that the problem concerned with conjunction-implicational paradoxes has had some historic discussion. For example, Nelson 1930 has objected to taking $(A\wedge B)\Rightarrow B$ as valid on the grounds that a portion of the antecedent is not relevant to the conclusion [1]. However, relevant logicians consider that there certainly is a sense of relevance between $A\wedge B$ and B, i.e., if any of the conjoined premises are used in arriving at the conclusion, then the conjoined premises are relevant to the conclusion [1].

Similar to the case of **CML**, for any formal theory $T_{\mathbf{T}}(P)$, $T_{\mathbf{E}}(P)$, or $T_{\mathbf{R}}(P)$, all conjunction-implicational and disjunction-implicational paradoxes are logical theorems of $T_{\mathbf{T}}(P)$, $T_{\mathbf{E}}(P)$, or $T_{\mathbf{R}}(P)$. As a result, for a conclusion of a reasoning from P based on **T**, **E**, or **R**, we cannot directly accept it as a valid conclusion in the sense of conditional, even if each of given premises is true. For example, from any given premise $A\Rightarrow B$, we can infer $(A\wedge C)\Rightarrow B$, $(A\wedge C\wedge D)\Rightarrow B$, and so on by using the logical theorem $(A\Rightarrow B)\Rightarrow((A\wedge C)\Rightarrow B)$ of **T**, **E**, and **R** and Modus Ponens for conditional, i.e., $(A\wedge C)\Rightarrow B\in T_{\mathbf{T/E/R}}(P)$, $(A\wedge C\wedge D)\Rightarrow B\in T_{\mathbf{T/E/R}}(P)$, … for any $A\Rightarrow B\in T_{\mathbf{T/E/R}}(P)$. However, from the viewpoint of scientific reasoning as well as our everyday reasoning, this reasoning is not necessarily regarded as valid in the sense of conditional because there may be no necessarily relevant and conditional relation between C, D, ... and B and therefore we cannot say "if A and C then B", "if A and C and D then B", and so on.

Therefore, because the traditional relevant logics accept conjunction-implicational and disjunction-implicational paradoxes as logical theorems, a reasoning based on the relevant logics still may be irrelevant.

In order to establish a satisfactory logic calculus of conditional to underlie relevant reasoning, the present author has proposed some

strong relevant logics, named **Rc**, **Ec**, and **Tc**, and shown their applications in knowledge engineering, in particular, scientific discovery [3-7]. As a modification of traditional relevant logics **R**, **E**, and **T**, strong relevant logics **Rc**, **Ec**, and **Tc** reject all conjunction-implicational paradoxes and disjunction-implicational paradoxes in **R**, **E**, and **T**, respectively. Therefore they are free not only of implicational paradoxes but also of conjunction-implicational and disjunction-implicational paradoxes. What underlies the strong relevant logics is the ***strong relevance principle***. We say that a logic system satisfies the ***strong relevance principle*** if for any logical theorem of the logic, say A, every sentential variable in A occurs at least once as an antecedent part and at least once as a consequent part. Those traditional relevant logics that only satisfy the relevance principle but not the strong relevance principle can be called the ***weak relevant logics***.

We can prove the following theorem [11].

Definition A is a consequent part of A; if ¬B is a consequent {antecedent} part of A, then B is an antecedent part {consequent part} of A; if B⇒C is a consequent {antecedent} part of A, then B is an antecedent {consequent} part of A, and C is a consequent {antecedent} part of A; if either B∧C or B∨C is a consequent {antecedent} part of A, then both B and C are consequent {antecedent} parts of A.

Theorem If A is a logical theorem of **Rc**, **Ec**, or **Tc**, then every sentential variable in A occurs at least once as an antecedent part and at least once as a consequent part.

Therefore, in the framework of strong relevant logics, the conclusion of a reasoning based on a strong relevant logic must be strongly relevant to its premises in the sense of the strong relevance principle. As a result, for such a conclusion, we can directly accept it as correct and do not need to investigate whether it is relevant to its premises or not.

5. Strong Relevance as a Logical Validity Criterion for Scientific Reasoning

The purpose of any scientific reasoning in any field is to find some both new and interesting facts, concepts, and principles from known facts or assumed hypotheses. The logical validity of reasoning is the only criterion that any scientific reasoning must act according to in order to obtain correct conclusions from the premises. It is a difficult task and open problem to formally define that the conclusion of a reasoning is new and interesting. But at least, as we have presented in Section 2, a scientific reasoning must be truth-preserving, relevant, ampliative, paracomplete, and paraconsistent.

The **CML** cannot underlie truth-preserving reasoning in the sense of conditional because in the framework of **CML**, even if a reasoning is valid in the sense of **CML**, the truth of its conclusion in the sense of conditional can be guaranteed necessarily. The truth-preserving property of a reasoning based on the **CML** is a matter of extensional truth-function but not in the sense of conditional, it is meaningless from the viewpoint of conditional or entailment.

The **CML** cannot underlie relevant reasoning because in the framework of **CML**, even if a reasoning is valid in the sense of **CML**, the necessary relevance between its premises and its conclusion can be guaranteed necessarily.

The **CML** cannot underlie ampliative reasoning because the conditional is represented in **CML** by the material implication which is no more than an extensional truth-function of its antecedent and consequent. As a result, a reasoning based on the logic must be circular. For example, Modus Ponens for material implication is usually represented in **CML** as "from A and A→B to infer B." According to the extensional truth-functional semantics of the material implication, if we know "A is true" but do not know the truth-value of B, then we cannot decide the truth-value of "A→B." In order to know the truth-value of B using Modus Ponens for material implication, we have to know the truth-value of B before the reasoning is done!

Reasoning with partial (and some time inconsistent) information is the rule rather than the exception in our real-life situations and most scientific disciplines. The **CML** cannot underlie paracomplete reasoning because it is a logic about tautologies of truth-function; it assumes that all the information and the truth-value of any sentence are on the table before any deduction is performed.

The **CML** cannot underlie paraconsistent reasoning because (¬A∧A)→B, which is a typical paradox of material implication so-called '***ex falso quodlibet***', is a logical theorem of it. As a result, in the framework of **CML**, reasoning under inconsistency is impossible because any formal theory $T_{CML}(P)$ must be explosive if it is directly or indirectly inconsistent. However, almost all formal theories based on empirical or experimental sciences generally may be indirectly inconsistent. This problem also exists in any classical conservative extension or non-classical alternative of **CML** where (A∧¬A)→B is accepted as a logical theorem and Modus Ponens for material implication serves as an inference rule.

Based on the above discussions, we can say that the classical validity criterion underlying the **CML** is only a necessary but not sufficient condition for scientific reasoning; it is not completely adequate to accomplishing the purpose of a scientific reasoning. In fact, the **CML** was established in order to provide formal languages for describing the structures with which mathematicians work, and the methods of proof available to them. It is not an original aim of the **CML** to underlie truth-preserving (in the sense of conditional), relevant, ampliative, paracomplete, and paraconsistent reasoning. It is obviously problematical to use the **CML** freely to those areas which exceed its defined application area.

As we have presented in Section 2, one of fundamental observations and assumptions on scientific discovery processes and their automation proposed by the present author is "Any automated scientific discovery process need an autonomous forward reasoning mechanism."

Any forward deduction system based on the **CML** or its any classical conservative extension must be ineffective and inefficient. In the framework of the **CML**, there is no guarantee that the conclusion of a reasoning is necessarily relevant to its premises, even if the reasoning is valid in the sense of the **CML**. This intrinsic characteristic of **CML** is a fatal defect for forward reasoning mechanism because any forward deduction system based on the **CML** must produce so many conclusions that are not relevant to premises at all.

On the other hand, relevant logics, in particular, strong relevant logics, can satisfactorily underlie truth-preserving (in the sense of conditional), relevant, ampliative, paracomplete, and paraconsistent reasoning, and forward deduction [3-7]. From the viewpoint to regard a logically valid reasoning as the process of drawing new and correct conclusions from some premises which are known facts or assumed hypotheses, we can say that any meaningful scientific reasoning required by a problem solving process in the real world must be relevant and ampliative. In order to accomplish the purpose of scientific reasoning, we should adopt not only the truth-preserving (in the sense of conditional) but also the strong relevance between conclusions and premises as a logical validity criterion for any scientific reasoning.

6. Concluding Remarks

We have pointed out why a reasoning based on the **CML**, its various classical conservative extensions, and traditional (weak) relevant logics may be irrelevant, and shown that a reasoning based on the strong relevant logics is relevant.

The fundamental logic underlying the Information Science, in particular, the Knowledge Science, in the 21st century should support relevant and ampliative reasoning as well as truth-preserving reasoning. Until now, the only family of logic to take relevance in reasoning into account is relevant logic. As a knowledge representation and reasoning tool, relevant logic, in particular, strong relevant logics, has many useful properties that the **CML** and its various conservative extensions do not have, and therefore, it is most hopeful candidate as the fundamental logic to underlie those advanced information/knowledge systems where the relevant reasoning plays a crucial role.

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