A Two-dimensional Modal Logic for Knowledge Representation in Asynchronous Multi-Agent Systems

Vania Costa Mário Benevides†
System Engineering and Computer Science Program, COPPE
†Mathematics Institute
Federal University of Rio de Janeiro
Mailbox 68511, zipcode 21945-970, Rio de Janeiro - RJ, Brazil
{vaniac,mario}@cos.ufrj.br

Abstract ***This paper introduces a two-dimensional modal logic to represent agents' knowledge in distributed environments. The agent's knowledge was formally defined in [5], where modal logic was used to model knowledge in synchronous distributed message-passing systems. The logic we present here can properly describe the properties of the agent's knowledge in asynchronous environments. An axiomatic system to describe such kind of knowledge is also presented.***

Keywords: multi-agent system; knowledge modal logic; multidimensional logic.

1 Introduction

The research in modal logic for knowledge representation has grown in the last two decades due to the work of J. Halpern, R. Fagin, Y. Moses and M. Vardi [5]. The formal systems of Halpern et al is mainly used to represent the knowledge of agents that communicate to each other through message passing in a distributed system. The logic enables one to represent the so-called *interactive knowledge*, that is, not only what an agent himself *knows* about the world, but also what he *knows* the others *know*.

The most complex knowledge interaction is the *common knowledge*. A fact ϕ is defined to be common knowledge in a group of agents if everybody knows, and everybody knows that everybody knows, and everybody knows that everybody knows that everybody knows, and this goes on indefinitely.

Common knowledge is a very intuitive concept but it is not a trivial task at all to formalize it. In the work of Halpern et al, it is proved that common knowledge requires coordinated actions and simultaneity to be attained. Hence, common knowledge can not be achieved in asynchronous systems, because simultaneity is not applicable in such environments.

If on the one hand the main application of Halpern et al knowledge logic is in the context of synchronous systems, where simultaneity is assumed, on the other hand we propose a logic to represent other concepts of knowledge that can be achieved in asynchronous environments. One of these such concepts is the *concurrent common knowledge*, defined in the paper by Panangaden and Taylor [8].

2 Model for Asynchronous Distributed Multi-Agent System

The model for asynchronous environments used here is based on Lamport's definitions of time and causality [6]: time is given by causality relations among events; and consistent global states are *consistent cuts* in an asynchronous run hypergraph.

Consider a simple model for an asynchronous distributed system [1]:

A network (*fifo* channels) with m agents;
A set R of asynchronous runs;
A set E of events;
A set C of consistent cuts.

The hypergraph in Figure 1 illustrates one possible run of the PIF (propagation of information with feedback) algorithm for 3 agents. The goal of PIF algorithm is to make the message $\mathcal{M}$ known to all the agents in the system, and, assuming that just one agent initiates the algorithm, to inform the initiator when $\mathcal{M}$ has reached all of them.

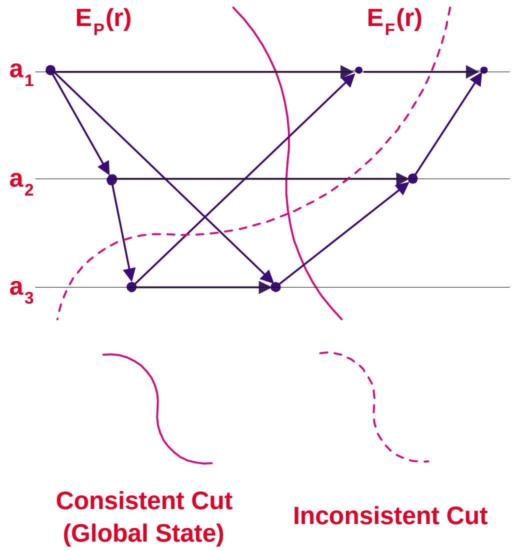

Figure 1: Consistent Cut

The dots represent **events** - when an agent sends and/or receives messages. The arrows establish a causality relation among events. A cut represents a global state and divides the graph into two sets of events, E_P and E_F, those which happen before (in the past of) and those which happen after (in the future of) the present cut. Intuitively, we can think about a **consistent cut** as a global state in which there are no messages from the future to the past.

[1]A complete description of a model for asynchronous distributed systems can be found in [1]

In this model, an agent can not distinguish between two cuts if his local state is the same in both cuts. If so, the cuts are said to be *indistinguishable* according to the agent's point of view. There are distinct possible runs depending on the order in which messages reach the agents - see figure 2.

3 Products of Modal Logics

We think about asynchronous systems as a two-dimensional world. That is, we reason about the knowledge under the perspective of a cartesian pair (r, c), a *run-cut* pair. In a modal logic approach, that means the interpretation of possible worlds, where the agent's knowledge is evaluated, are pairs (r, c) representing a state: a consistent cut c in an asynchronous run r.

The two-dimensional approach of knowledge in asynchronous multi-agent distributed systems can be formalized using the concept of *products of modal logics*. Many-dimensional or multidimensional logics are a kind of cartesian product of modal logics. In multidimensional logics, the possible worlds, or states, are tuples representing dimensions where logical formulas are evaluated. The foundations of multidimensional logics are found in Segerberg [9].

Products of modal logics are formally defined in [10]. Let L be a set of formulas and $\mathbf{F}(L)$ the class of frames validating L, that is, for all wff $\alpha \in L$, α is valid in each frame of the class $\mathbf{F}$. Conversely, all formulas valid in a certain frame F constitute the modal logic $\mathbf{L}(F)$. The modal logic $\mathbf{L}(\mathbf{F})$ for a class of frames $\mathbf{F}$ is defined as the intersection $\bigcap\{\mathbf{L}(F) \mid F \in \mathbf{F}\}$.

Definition 3.1 *Products of Modal Logics.*

Let $F_1 = (W_1, \cong_i)$ *and* $F_2 = (W_2, \asymp_j)$ *be two propositional frames. The* product of frames *[10] is the frame* $F_1 \times F_2 = (W_1 \times W_2, \simeq_i, \approx_j)$*, where:*

$\simeq_i = \{((x, z), (y, z)) | x \cong_i y\}$;
$\approx_j = \{((z, x), (z, y)) | x \asymp_j y\}$.

Let L_1 *and* L_2 *be multimodal logics,* $\mathbf{F}(L_1)$ *the class of frames validating* L_1 *and* $\mathbf{F}(L_2)$

the class of frames validating L_2. *The product of logics* L_1 *and* L_2 *is the logic* $L_1 \times L_2 = \mathbf{L}(\mathbf{F}(L_1) \times \mathbf{F}(L_2))$.

Results from V. Shehtman's and D. Gabbay's paper [10] on axiomatizing products of modal logics follow.

Definition 3.2 *Commutative Logics.*

For L_1 *n-modal and* L_2 *m-modal logics:*
$[L_1, L_2] = L_1 * L_2 + C^1_{ij} + C^2_{ij}$, *where:*
$C^1_{ij} = (\Box_i \Box_{j+n} p \leftrightarrow \Box_{j+n} \Box_i p)$,
$C^2_{ij} = (\Diamond_i \Box_{j+n} p \rightarrow \Box_{j+n} \Diamond_i p)$,
$1 \leq i \leq n \ , \ 1 \leq j \leq m$.

We say the logics L_1, L_2 are *commutatives* if $L_1 \times L_2 = [L_1, L_2]$.

Definition 3.3 *PTC Formulas and PTC Logics.*

A modal formula is pseudo-transitive if it has the form:

$\nabla_1 \Box_k p \rightarrow \triangle_2 p$, *where* $p \in Prop$, $\nabla_1 = \Diamond_i, \ldots, \Diamond_j$, $\triangle_2 = \Box_i, \ldots, \Box_j$ *are sequences of modal operators (possibly empty).*

A PTC formula is a pseudo-transitive or closed formula.

A PTC logic is a modal logic axiomatized by PTC formulas.

Theorem 3.1 *Axiomatization of the Product of PTC Modal Logics [10].*

The logic resulting from the product of two PTC (pseudo-transitive or closed) modal logics is commutative.

That is, if L_1 , L_2 *are PTC then* $L_1 \times L_2 = [L_1, L_2]$.

4 Semantics for Knowledge in Asynchronous Systems

To model the desired two-dimensional knowledge approach we need a two-dimensional multimodal logic. Thus, the dimensions considered are runs and cuts.

Once this semantics is based on Kripkean semantics of possible worlds, we have possibility or accessibility relations. There are three possibility relations, each one respectively associated to a modal operator. These relations are equivalence relations, reflecting the concept of indistinguishable cuts or runs according to the agent's point of view. Hence, the possibility relations are, in fact, equivalence relations for indistinguishability in each level of knowledge considered: the run dimension, the cut dimension, and the run-cut dimension.

Walking in the run dimension would give us the possible consistent cuts of that run. In our modal product, runs are in the horizontal axis - the corresponding modal operator is H_i, and the indistinguishable cuts according to the agent's point of view is given by the horizontal relation $\simeq_i$.

On the other hand, walking on the cut dimension would correspond to having the runs where a consistent cut occurs. Thus, consistent cuts are represented in the vertical axis, and the corresponding modal operator is V_i. The indistinguishability vertical relation $\approx_i$ refers to the indistinguishable runs for the agent, taking a particular cut in consideration.

We introduce the definition of *closed sub-product of modal logics* in order to formalize the kind of knowledge that we are interested in. The closed sub-product of modal logics is similar to the former product, with two additional features: an extra relation, the reflexive-transitive-symmetric closure under the two basic relations, and a subset A of the cartesian product $X \times Y$.

The closure relation gives us new features: for instance, representing knowledge properties according to indistinguishable pairs (r, c). The modal operator K_i is related to the indistinguishability inter-dimensional relation $\sim_i$, and represents what an agent knows under indistinguishable consistent cuts in all possible runs.

There are some pairs (r, c) that, in fact, may not occur in the system. If so, we restricted the evaluation of the formulas to what we call the *reasonable* pairs, that is, the pairs (r, c) that really make sense. We define the subset $A \subseteq X \times Y$ denoting these reasonable pairs. The idea is to make the modal operators H_i and V_i range only over the reasonable pairs in

A, whereas the operators $\overline{H}_i$ and $\overline{V}_i$ range over the whole cartesian product $W = X \times Y$.

Definition 4.1 *Closed Sub-product of Modal Logics.*

Consider the propositional frames $F_1 = (X, \cong_i)$ *and* $F_2 = (Y, \asymp_i)$ *for* L_1 *and* L_2*. The* closed sub-product *of the frames is the frame* $F_1 \otimes F_2 = (W, \simeq_i, \approx_i, \sim_i, A)$*, where:*

1. $W = X \times Y$ *: is the set of all states* (x, y)*;*
2. $A \subseteq W = X \times Y$ *: is a subset of states* (x, y)*;*
3. $\simeq_i = \{((x, z), (y, z)) | x \cong_i y\}$*;*
4. $\approx_i = \{((z, x), (z, y)) | x \asymp_i y\}$*;*
5. $\sim_i \subseteq \{(\simeq_i + \approx_i)^*\}$, where $(\simeq_i + \approx_i)^*$ denotes the reflexive-transitive-symmetric closure under the union of $\simeq_i$ and $\approx_i$;

Let L_1 *and* L_2 *be multimodal logics,* $\mathbf{F}(L_1)$ *the class of frames validating* L_1 *and* $\mathbf{F}(L_2)$ *the class of frames validating* L_2*. The closed sub-product of logics* L_1 *and* L_2 *is the logic* $L^2_m = \mathbf{L}(\mathbf{F}(L_1) \otimes \mathbf{F}(L_2))$*.*

Definition 4.2 *Model for Closed Sub-product of Modal Logics.*

Consider L_1 *and* L_2 *multimodal logics with sets of primitives* $Prop_1$ *and* $Prop_2$*,* $F_1 \in \mathbf{F}(L_1)$ *and* $F_2 \in \mathbf{F}(L_2)$*, respectively. A* model M *over* $F = F_1 \otimes F_2$ *is a pair* $M = (F, v)$*, where* $v : Prop \rightarrow 2^W$ *is an assignment function,* $Prop = Prop_1 \cup Prop_2$*. For each* $p \in Prop = Prop_1 \cup Prop_2$*,* $v(p)$ *is the set of pairs* $(x, y), (x, y) \in W = X \times Y$*, where* p *is true.*

Many known modal logics are PTC, such as $\mathcal{D}, \mathcal{K}4, \mathcal{S}4, \mathcal{T}, \mathcal{B}, \mathcal{S}5$, and others. Hence, two-dimensional products such as $\mathcal{T} \times \mathcal{T}$, $\mathcal{S}4 \times \mathcal{S}4$, $\mathcal{S}5 \times \mathcal{S}5$ are *commutatives*.

Our main interest is in $\mathcal{S}5 \times \mathcal{S}5$. As shown in [5], a modal logic for knowledge, with an equivalence relation as the accessibility relation, can be axiomatized by system $\mathcal{S}5_m$ with m modal operators. We define the axiomatic system $\mathcal{S}^2_m$ for the two-dimensional knowledge logic as an extension of the product $\mathcal{S}5_m \times \mathcal{S}5_m$.

Another important semantic consideration is the meaning of knowledge in the K_i modal operator. In general, knowledge is associated with the agent's local state. For instance, in [8], the agent's local state is defined as his local history. In our semantics, the knowledge interpretation is associated with the agent's *past view* - all events in his local past history, regardless the order they happened.

Formal definitions of the two-dimensional semantics follow.

Definition 4.3 *Two-Dimensional Logic* L^2_m*.*

Let L^2_m *be the smallest set of formulas containing* Δ*, the set of primitives* $Prop = Prop_H \cup Prop_V$*, closed under negation, conjunction and the modal operators* $\overline{H}_i$*,* $\overline{V}_i$ *and* K_i*, where* $i = 1, \ldots, m$*.*

Definition 4.4 *Satisfiability in* L^2_m*.*

Suppose $\simeq_i$*,* $\approx_i$ *and* $\sim_i$ *are equivalence relations, referred as indistinguishability relations, between two states* (r, c) *and* (r', c') *in a closed sub-product of two modal frames, as defined in 4.1.*

Let $F = (W, \simeq_i, \approx_i, \sim_i, A)$ *be a frame for* L^2_m *and let* M *be a model over* F*. A formula* $\alpha \in L^2_m$ *is true in* $[M, (r, c)]$*,* $[M, (r, c)] \models \alpha$*, for* $(r, c) \in W \subseteq R \times C$*, when:*

1. $[M, (r, c)] \models p \Leftrightarrow (r, c) \in v(p)$*, where* $p \in Prop$*;*
2. $[M, (r, c)] \models \alpha \wedge \beta \Leftrightarrow [M, (r, c)] \models \alpha$ *and* $[M, (r, c))] \models \beta$*;*
3. $[M, (r, c)] \models \neg\alpha \Leftrightarrow [M, (r, c)] \not\models \alpha$*;*
4. $[M, (r, c)] \models \overline{H}_i\alpha \Leftrightarrow \forall(r', c')\{((r, c) \simeq_i (r', c')) \Rightarrow [M, (r', c')] \models \alpha\}$*;*
5. $[M, (r, c)] \models \overline{V}_i\alpha \Leftrightarrow \forall(r', c')\{((r, c) \approx_i (r', c')) \Rightarrow [M, (r', c')] \models \alpha\}$*;*
6. $[M, (r, c)] \models \Delta \Leftrightarrow (r, c) \in A \subseteq W = R \times C$*;*
7. $[M, (r, c)] \models H_i\alpha \Leftrightarrow [M, (r, c)] \models \Delta$ *and* $[M, (r, c)] \models \overline{H}_i\alpha$*;*

8. $[M,(r,c)] \models V_i\alpha \Leftrightarrow [M,(r,c)] \models \Delta$ *and* $[M,(r,c)] \models \overline{V}_i\alpha$;

9. $[M,(r,c)] \models Q_i\alpha \Leftrightarrow [M,(r,c)] \models H_i\alpha$ *and* $[M,(r,c)] \models V_i\alpha$;

10. $[M,(r,c)] \models K_i\alpha \Leftrightarrow [M,(r,c)] \models \Delta$ *and* $\forall(r',c')\{((r,c) \sim_i (r',c')) \Rightarrow [M,(r',c')] \models \alpha\}$.

5 Axiomatic System for Two-Dimensional Knowledge

The two-dimensional multimodal system $\mathcal{S}^2_m$ has three basic modalities. The modalities H_i, V_i and K_i refer respectively to the properties of the horizontal, vertical and two-dimensional worlds. Intuitively, H_i, V_i and K_i represent, respectively, the knowledge in the run dimension, in the cut dimension and according to the two-dimensional run-cut perspective. In fact, we use the knowledge representation semantics because it was the initial motivation, but we think the theory has possibly many other interpretations.

The axiomatic system $\mathcal{S}^2_m$ follows.

Axioms.

0 All the tautologies from propositional logic

1 $(\overline{H}_i\alpha \wedge \overline{H}_i(\alpha \rightarrow \beta)) \rightarrow \overline{H}_i\beta$

2 $\overline{H}_i\alpha \rightarrow \alpha$

3 $\overline{H}_i\alpha \rightarrow \overline{H}_i\overline{H}_i\alpha$

4 $\neg\overline{H}_i\alpha \rightarrow \overline{H}_i\neg\overline{H}_i\alpha$

5 $(\overline{V}_i\alpha \wedge \overline{V}_i(\alpha \rightarrow \beta)) \rightarrow \overline{V}_i\beta$

6 $\overline{V}_i\alpha \rightarrow \alpha$

7 $\overline{V}_i\alpha \rightarrow \overline{V}_i\overline{V}_i\alpha$

8 $\neg\overline{V}_i\alpha \rightarrow \overline{V}_i\neg\overline{V}_i\alpha$

9 $(K_i\alpha \wedge K_i(\alpha \rightarrow \beta)) \rightarrow K_i\beta$

10 $K_i\alpha \rightarrow \alpha$

11 $K_i\alpha \rightarrow K_iK_i\alpha$ [2]

12 $\neg K_i\alpha \rightarrow K_i\neg K_i\alpha$

13 $\overline{H}_i\overline{V}_j\alpha \leftrightarrow \overline{V}_j\overline{H}_i\alpha$

14 $\neg\overline{H}_i\neg\overline{V}_j\alpha \rightarrow \overline{V}_j\neg\overline{H}_i\neg\alpha$

15 $\neg\overline{V}_i\neg\overline{H}_j\alpha \rightarrow \overline{H}_j\neg\overline{V}_i\neg\alpha$

16 $H_i\alpha \leftrightarrow \Delta \wedge \overline{H}_i\alpha$

17 $V_i\alpha \leftrightarrow \Delta \wedge \overline{V}_i\alpha$

18 $Q_i\alpha \leftrightarrow H_i\alpha \wedge V_i\alpha$

19 $K_i\alpha \leftrightarrow Q_iK_i\alpha$

20 $K_i(\alpha \rightarrow Q_i\alpha) \rightarrow (\alpha \rightarrow K_i\alpha)$

where $i, j = 1, \ldots, m$.

Rules.

R0 From $\vdash \alpha$ infer all uniform substitution

R1 From $\vdash \alpha, \alpha \rightarrow \beta$ derive β (modus ponens)

R2 From $\vdash \alpha$ infer $\overline{H}_i\alpha$ (*horizontal generalization*)

R3 From $\vdash \alpha$ infer $\overline{V}_i\alpha$ (*vertical generalization*)

R4 From $\vdash \alpha$ infer $K_i\alpha$ (*two-dimensional generalization*)

In such system, knowledge is also defined in terms of fixed point formulas. Although, knowledge refers to a bidimensional approach, representing what and agent knows in all indistinguishable states (r, c). We have soundness and completeness proofs for system $\mathcal{S}^2_m$. For completeness, we can show the finite model property, and then we have, in addition, decidability.

6 Example

An example to illustrate knowledge according to the two-dimensional approach was built. It is an application of PIF (propagation of information with feedback) distributed algorithm [1]. The example considers three agents executing the PIF algorithm with one initiator and *fifo* reliable channels linking the three agents.

[2]this axiom can be obtained from 19 and 20.

At first, we define the set E of events to characterize message exchanging among the agents. In our model, the events have the same definition regardless the run they can occur.

Suppose $send_i^j(\mathcal{M})$ means "agent i sends message $\mathcal{M}$ to agent j", and $receive_i^j(\mathcal{M})$ means "agent i receives message $\mathcal{M}$ from agent j". Let $E = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7\}$, where:

e_1 - $send_1^2(\mathcal{M})$, $send_1^3(\mathcal{M})$
e_2 - $receive_2^1(\mathcal{M})$, $send_2^3(\mathcal{M})$
e_3 - $receive_3^2(\mathcal{M})$, $send_3^1(\mathcal{M})$
e_4 - $receive_3^1(\mathcal{M})$, $send_3^2(\mathcal{M})$
e_5 - $receive_2^3(\mathcal{M})$, $send_2^1(\mathcal{M})$
e_6 - $receive_1^3(\mathcal{M})$
e_7 - $receive_1^2(\mathcal{M})$

We built the sets E_i, $i = 1, 2, 3$, where E_i represents the possible events for agent i:

$E_1 = \{e_1, e_6, e_7\}$
$E_2 = \{e_2, e_5\}$
$E_3 = \{e_3, e_4\}$

Enumerating the possible runs for the PIF algorithm we get the set $R = \{r_1, r_2, r_3, r_4, r_5, r_6\}$. See figure 2 for three of the six runs.

The possible consistent cuts in all runs in R were also enumerated. Note that each cut is a partition of the events in E into two sets E_P and E_F. It follows the enumeration for each of the sixteen cuts in the set C. [3]:

c_1: $E_P = \{e_1\}$; $E_F = \{e_2, e_3, e_4, e_5, e_6, e_7\}$
c_2: $E_P = \{e_1, e_2\}$; $E_F = \{e_3, e_4, e_5, e_6, e_7\}$
c_3: $E_P = \{e_1, e_2, e_3\}$; $E_F = \{e_4, e_5, e_6, e_7\}$
c_4: $E_P = \{e_1, e_2, e_3, e_4\}$; $E_F = \{e_5, e_6, e_7\}$
c_5: $E_P = \{e_1, e_2, e_3, e_4, e_5\}$; $E_F = \{e_6, e_7\}$
c_6: $E_P = \{e_1, e_2, e_3, e_4, e_5, e_6\}$; $E_F = \{e_7\}$
c_7: $E_P = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7\}$; $E_F = \{\}$
c_8: $E_P = \{e_1, e_2, e_3, e_6\}$; $E_F = \{e_4, e_5, e_7\}$
c_9: $E_P = \{e_1, e_2, e_3, e_6, e_4\}$; $E_F = \{e_5, e_7\}$
c_{10}: $E_P = \{e_1, e_2, e_3, e_4, e_5, e_7\}$; $E_F = \{e_6\}$
c_{11}: $E_P = \{e_1, e_4\}$; $E_F = \{e_2, e_3, e_5, e_6, e_7\}$
c_{12}: $E_P = \{e_1, e_2, e_4, e_5\}$; $E_F = \{e_3, e_6, e_7\}$
c_{13}: $E_P = \{e_1, e_2, e_4\}$; $E_F = \{e_3, e_5, e_6, e_7\}$
c_{14}: $E_P = \{e_1, e_2, e_4, e_5, e_7\}$; $E_F = \{e_3, e_6\}$
c_{15}: $E_P = \{e_1, e_4, e_5\}$; $E_F = \{e_2, e_3, e_6, e_7\}$
c_{16}: $E_P = \{e_1, e_4, e_5, e_7\}$; $E_F = \{e_2, e_3, e_6\}$

For agent 1, consider that knowledge is associated to the agent's past view. Thus, we have the following sets of past visions $V_P(c_i^1)$ for agent 1 in cuts i:

$V_P(c_i^1) = \{e_1\}$, $i = 1, 2, 3, 4, 5, 11, 12, 13, 15$;
$V_P(c_j^1) = \{e_1, e_6\}$, $j = 6, 8, 9$;
$V_P(c_k^1) = \{e_1, e_7\}$, $k = 10, 14, 16$;
$V_P(c_l^1) = \{e_1, e_6, e_7\}$, $l = 7$.

The accessibility relation says that two cuts are related if they are indistinguishable according to the agent's point of view, that is, if the agent has the same past view in both cuts. If so, the sets i, j, k, l of cuts listed above are equivalence classes representing knowledge states for the agent. Thus, in each of the sets i, j, k, l the agent *knows* the same things. We built graphs, one for each agent, mapping these equivalence classes of knowledge. Figure 3 is a reflexive-transitive reduction graph (that is, no reflexive-transitive links were represented) of the equivalence classes for agent 1.

7 Conclusions

The axiomatic system S_m^2 is suitable to represent the properties of an agent's knowledge and group knowledge in asynchronous systems because the semantics is based on a model which considers consistent cuts and asynchronous runs to define time in multi-agent distributed environments.

We have used the concept of multidimensional logics to deal with the two-dimensional approach of knowledge. Thus, it is possible to describe properties of each projected dimension - runs and cuts - and also the features of the product dimension. The closed subproduct of logics was defined to make the necessary adjustments. Hence, the resulting semantics results in a more powerful one.

As future developments, we would like to build a temporal version of the two-dimensional knowledge logic, which would better describe the evolution of knowledge acquisition over time.

[3] The cut c_0: $E_P = \{\}$; $E_F = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7\}$ corresponding to the initial state was not considered.

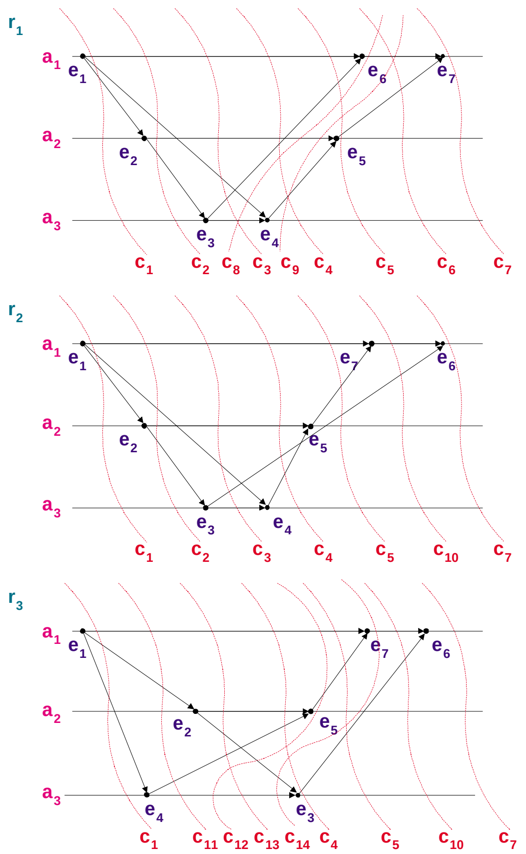

Figure 2: PIF Algorithm with 3 Agents - Asynchronous Runs r_1, r_2, r_3

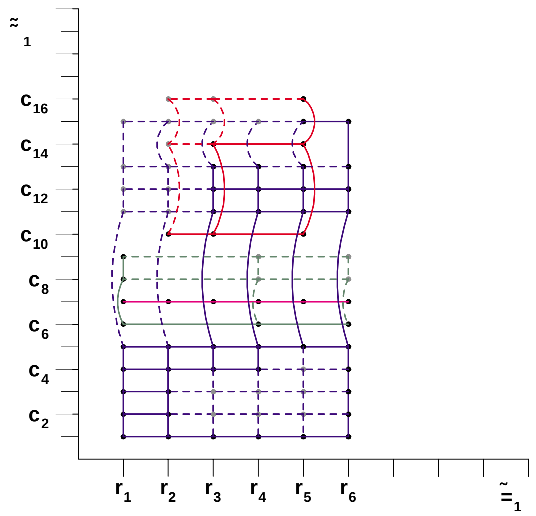

Figure 3: Knowledge Equivalence Classes for Agent 1

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