

Semantics for Default Reasoning

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Abstract. We present a bi-valued semantics for default logic appealing to maximal sets, instead of a fixed point of an operator as it is done by its creator Raymond Reiter. It has the advantage of precluding incoherent theories (theories with no extension), and it fixes many drawbacks of Reiter's original formalism. This approach is then generalized, using a many-valued logic (the logic FOUR of Belnap) as the underlining semantics, to construct a paraconsistent and nonmonotonic logic. Besides, we introduce a new notation for defaults which represents them as formulae and not as (pseudo) inference rules. The resulting formalism is both simple and powerful.

Key Words: Nonmonotonic reasoning, default logic, paraconsistent logic, semantics.

1 Some drawbacks in default logic

Default logic has been introduced by Raymond Reiter in 1980 [2]. Along the years it has become the most popular nonmonotonic formalism in Artificial Intelligence. Reiter introduces a *default* as an inference rule which allows the derivation of a proposition from absence of information. Let L be a first-order language as usually defined. The general form of a default is $\alpha : \beta_1, \dots, \beta_n / \beta$, where first-order logic formula α is called the *prerequisite*, β_1, \dots, β_n , for $n \geq 1$, are called the *justification* and β is called the *consequent*. *Seminormal* defaults are defaults of the form $\alpha : \beta, \beta_1 / \beta$, where the first justification and the consequent coincide. A default of the form $\alpha : \beta / \beta$ is called a *normal* default. A default is closed if all its component formulae are closed. A default theory is a pair $\langle W, D \rangle$ where W is a set of

formulae and D is a set of defaults. A default theory is closed if both sets are composed of closed expressions only. For any set of formulae S define $Th(S) = \{\alpha \in L ; S \vdash \alpha\}$, where (\vdash) denotes classical logic derivation.

Let $\langle W, D \rangle$ be a default theory, Reiter defines extension, the set of theorems from $\langle W, D \rangle$, as follows. For any set $S \subseteq L$, let $\Gamma(S)$ be the smallest set satisfying the following three properties:

- (1) $W \subseteq \Gamma(S)$
- (2) $Th(\Gamma(S)) = \Gamma(S)$
- (3) If $\alpha : \beta_1, \dots, \beta_n / \beta \in D$ and $\alpha \in \Gamma(S)$, and $\neg\beta_1, \dots, \neg\beta_n \notin S$, then $\beta \in \Gamma(S)$.

A set of closed formulae $E \subseteq L$ is an *extension* for $\langle W, D \rangle$ iff $\Gamma(E) = E$. That is, E is a fixed point of the operator Γ .

In spite of the great success and popularity default logic achieved in its 20 years of existence, there are several drawbacks in Reiter's original formulation in need to be fixed.

- *Language.* Defaults as presented by Reiter, are not *real* inference rules, since the derivation of the conclusion depends upon the non-derivation of the justification, and non-derivation cannot be established locally, it depends on verifying everything derived from the whole theory, a rather untractable and indeed undecidable problem. Nonmonotonicity and undecidability of default logic arise exactly from this feature. Considering a default as just another formula could be much simpler and natural.
- *Meaningless defaults.* Prerequisite, multiple justification and arbitrary defaults are totally dispensable and their use is misleading and

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counter-intuitive. Only prerequisite free normal and seminormal defaults are necessary.

- *Extensions.* The definition of extension as a fixed point is non-constructive and leads to undesired consequences. Any automatization of default logic must avoid the fixed point.
- *Incoherence.* In consequence of extensions being fixed points some theories have no extension.
- *Anomalous extensions.* Probably the most serious drawback of default logic and the other seminal logics presented in [3], it has been pointed out by Hanks and McDermott [4] back in 1987. Generating unexpected extensions, default logic draws conclusions not vindicated by common sense, jeopardizing its soundness.
- *No semantics.* Default logic has been introduced by Reiter syntactically, via pseudo inference rules, and no later attempt to provide semantics to default logic has indeed succeeded [5, 6, 7].

The maximal default logic presented next fixes all these drawbacks. Furthermore, the semantics proposed for it allows generalization for paraconsistent and multi-valued semantics as it is done in section 4.

2 Maximal default logic

We present a default logic called, maximal default logic, that fixes all drawbacks mentioned in the foregoing section. A more complete presentation of maximal default logic is done in [1].

2.1 Language

We adopt classical first-order logic language L augmented with a new connective (∞) for representing defaults. Defaults are expressions of the form $\beta \infty \gamma$, where β and γ are first-order formulae and γ is optional. β is called the *conclusion* and γ the *exception*. $\beta \infty \gamma$ reads as “ β unless γ ”. The semantical intuition behind it is that the non-satisfaction of γ implies (and it is implied by) the satisfaction of β . The

syntactical intuition is that β should be inferred if it is consistent (with the extension) and γ is not inferred (in the extension). A default theory is a pair $\langle W, D \rangle$, where W is a set of first-order formulae and D is a set of defaults. A default is *closed* if its component formulae are closed. A default theory is closed if both sets are composed of closed expressions only. Henceforth all default theories are closed. Let S be a set of defaults, define $CONS(S) = \{\beta; \beta \infty \gamma \in S\}$.

2.2 No meaningless defaults

It is easy to see that the default formula $\beta \infty \gamma$ corresponds to Reiter seminormal default $:\beta, \neg\gamma/\beta$ and default formula $\beta \infty$ corresponds to normal default $:\beta/\beta$. These are the only defaults that are necessary for nonmonotonic reasoning and defaults with prerequisites, multiple justifications and arbitrary defaults of any sort are dispensable.

2.3 Extensions

In order to define extensions for a default theory, the set of theorems, Reiter appeals to a fixed point of an operator on sets of formulae. This is non-constructive and causes some theories to have no extension. In our approach an extension is determined by a maximal subset of defaults, according to definition 1.

Definition 1. Let $\langle W, D \rangle$ be a default theory. A set $S \subseteq D$ is a *generating set* of an extension iff S is a maximal set of defaults such that

1. $W \cup CONS(S)$ is consistent;
2. If $\beta \infty \gamma \in S$ then $\gamma \notin Th(W \cup CONS(S))$.

S maximal with respect to properties (1) and (2) means that for all $S' \subseteq D$, such that (1) and (2) hold for S' and $S \subseteq S'$, then $S = S'$. $E = Th(W \cup CONS(S))$ is the *maximal extension* for $\langle W, D \rangle$.

2.4 No incoherent theories

Theorem 1 assures that all default theories $\langle W, D \rangle$ have a maximal extension (different from L as long as W is consistent). The proof is just an adaptation of Lindenbaum theorem that asserts that any consistent set of first-order formulae can be extended to a maximal consistent set.

Theorem 1. Let $\langle W, D \rangle$ be a default theory where W is consistent, then there is a maximal extension for $\langle W, D \rangle$.

Theorem 2 shows that our definition generalizes the one given by Reiter since all extensions according to Reiter's definition are maximal extensions, but the converse is not true, as it is shown by the incoherent theories.

Theorem 2. Let $\langle W, D \rangle$ be a default theory and $D' = \{:\beta, \neg\gamma / \beta; \beta \in D\}$. If E is an extension for $\langle W, D' \rangle$ according to Reiter then E is a maximal extension for $\langle W, D \rangle$.

Example 1.

$$\begin{aligned} W &= \emptyset \\ D &= \{A \in B, B \in C, C \in A\} \\ D' &= \{:A, \neg B / A, :B, \neg C / B, :C, \neg A / C\} \end{aligned}$$

The theory $\langle W, D \rangle$ has three maximal extensions, namely: $E_1 = Th(A)$, $E_2 = Th(B)$ and $E_3 = Th(C)$. However, the theory $\langle W, D' \rangle$ has no extension according to Reiter.

2.5 No anomalous extensions

Anomalous extensions in default logic arise whenever the exception to a default is itself derived by (another) default. The two defaults are, then, incompatible, in the sense that they cannot be generating defaults to a same extension. Therefore, they are split in two extensions, one containing the first default and the other containing the exception. Common sense reasoning adheres to the latter extension, and the former is considered to be unintuitive or anomalous.

Let us state the problem in the formalized language.

Example 2. The pattern¹.

$$\begin{aligned} W &= \{C \wedge A\} \\ D &= \{(A \rightarrow \neg F) \in B, (B \rightarrow F) \in, (C \rightarrow B) \in\} \end{aligned}$$

Notice that B is indeed an exception to the fact that A implies $\neg F$ not only because it bars the derivation of the implication but also because it implies F . This theory has two (maximal) extensions namely:

$$\begin{aligned} E_1 &= Th(C \wedge A, B \rightarrow F, C \rightarrow B) \text{ and} \\ E_2 &= Th(C \wedge A, B \rightarrow F, A \rightarrow \neg F) \end{aligned}$$

E_1 is the extension vindicated by common sense while E_2 is an anomalous extension.

The principle that states that the derivation of an exception has priority over the derivation of the default to which it is an exception, the situation on pattern of example 2, has been called in [9] the *exceptions-first* principle. It is a very general and formal (it does not depends on the semantics of the theory) principle. It applies to all contexts independently on the subject matter, might it be temporal projection, causal relations, frame problem, or any other sort of epistemic situation which requires practical or common sense reasoning.

To implement the exceptions-first principle an ordering on defaults is introduced in which a default is lesser than another, if it relevant (in the context of the theory) to the derivation of an exception to the latter. The set of generating defaults of an extension is then built respecting this ordering, i.e., defaults of higher order cannot rule out from the extension a default lesser than it.

Definition 2. Let $\langle W, D \rangle$ be a default theory, α a formula and $R \subseteq D$. R is a support set for α iff:

1. $W \cup CONS(R)$ is consistent;
2. $\alpha \in Th(W \cup CONS(R))$;

¹ The anomalous pattern appeared for the first time in [8] back to 1989.

3. R is minimal with respect to \subseteq , i.e., if there is $R' \subseteq R$ which satisfies conditions 1 and 2, then $R = R'$.

R is intended to be a minimal set of defaults which is used to prove α . Notice that a formula may have several associated support sets.

Definition 3. Let $\langle W, D \rangle$ be a default theory, d_1, d_2 and d_3 be defaults in D and α be a formula. Then:

1. $d_1 \ll \alpha$ iff $d_1 \in R$, for some support set R of α ;
2. $d_1 \prec_i d_2$ iff d_2 is of the form $\beta \propto \alpha$ and $d_1 \ll \alpha$;
3. $d_1 \prec d_2$ iff (a) $d_1 \prec_i d_2$ or (b) there is a default d_3 such that $d_1 \prec d_3$ and $d_3 \prec d_2$.

Intuitively $d \ll \alpha$ implies that the conclusion of d may be used to prove α ; $d_1 \prec_i d_2$ means that the conclusion of d_1 may be used to prove the exception of d_2 ; and relation \prec is the transitive closure of \prec_i .

Definition 4. A default theory $\langle W, D \rangle$ is *acyclic* iff there is no default $d \in D$ such that $d \prec d$. Hence \prec is a strict order among defaults of an acyclic theory.

The theory in example 1 is cyclic, while the theory in example 2 is acyclic, it is not coincidence that in example 1 the default theory has no extension. Etherington proved in [5] that to be acyclic is a sufficient, but not necessary, condition for a default theory to have an extension in Reiter's sense. Our definition of maximal extension complying with the exceptions-first principle applies only to acyclic theories, since it requires the order \prec to be strict.

Definition 5. Let $\langle W, D \rangle$ be an acyclic default theory. A set $S \subseteq D$ is a *generating set* of an *extension complying with the exceptions-first principle* iff S is a maximal set of defaults such that

1. $W \cup \text{CONS}(S)$ is consistent;
2. If $\beta \propto \gamma \in S$, then $\gamma \notin \text{Th}(W \cup \text{CONS}(S))$;
3. For all $d \in S$, if d' is of the form $\beta \propto \gamma$ and $d' \prec d$ and $d' \notin S$, then β is inconsistent with

$\text{Th}(W \cup S_{\prec d})$, or $\gamma \in \text{Th}(W \cup S_{\prec d})$, where $S_{\prec d} = \{d \in S; d' \not\prec d\}$.

S is maximal with respect to properties (1), (2) and (3) means that for all $S' \subseteq D$, such that (1), (2) and (3) hold for S' and $S \subseteq S'$, then $S = S'$. $E = \text{Th}(W \cup \text{CONS}(S))$ is the maximal extension for $\langle W, D \rangle$ complying with the exceptions-first principle.

Example 2 revisited.

$W = \{C \wedge A\}$
 $D = \{(A \rightarrow \neg F) \propto B, (B \rightarrow F) \propto, (C \rightarrow B) \propto\}$

Notice that $(C \rightarrow B) \propto \prec (A \rightarrow \neg F) \propto B$ and this theory is acyclic. Therefore $E = \text{Th}(C \wedge A, B \rightarrow F, C \rightarrow B)$ is the only maximal extension complying with the exceptions-first principle.

3 Semantics for maximal default logic

Let I be a first-order logic interpretation. Satisfaction of a formula φ ($I \models \varphi$) of a set of formulae Γ and the consequence relation ($\Gamma \models \varphi$) is defined as in classical logic. I satisfies a default $\beta \propto \gamma$ ($I \models \beta \propto \gamma$) iff $I \models \beta$ and $I \not\models \gamma$. Given a default theory $\langle W, D \rangle$ and an interpretation I , define $I_D = \{\beta \propto \gamma \in D; I \models \beta \propto \gamma\}$. I satisfies a default theory $\langle W, D \rangle$ ($I \models \langle W, D \rangle$) iff $I \models W$ and I_D is maximal, in the sense that, for all I' such that $I' \models W$ and $I_D \subseteq I'_D$, then $I_D = I'_D$. I is called a *maximal interpretation* for $\langle W, D \rangle$. Notice that our notion of interpretation for default theory appeals to maximality whereas the corresponding notion of extension in default logic uses a fixed point. As a matter of fact, in [1] we show that maximal interpretation is indeed the semantical counterpart of extension, since the generating defaults of an extension are satisfied by a maximal interpretation and vice-versa.

A default theory $\langle W, D \rangle$ *skeptically entails* a formula α ($\langle W, D \rangle \models_s \alpha$) iff for all I such that $I \models \langle W, D \rangle$, $I \models \alpha$. $\langle W, D \rangle$ *credulously entails* a formula α ($\langle W, D \rangle \models_c \alpha$) iff there

exists I such that $I \models \langle W, D \rangle$ and for all I' such that $I' \models \langle W, D \rangle$ and $I'_d = I_d$, then $I' \models \alpha$.

Example 3.

$$W = \{B, P \rightarrow B\}$$

$$D = \{(B \rightarrow F) \propto P\}$$

For intuition, consider B stands for *Bird*, P stands for *Penguin*, F stands for *Fly*. Any interpretation I such that $I \models B$, $I \models F$, and $I \not\models P$, satisfies $\langle W, D \rangle$. Notice that if an interpretation I is maximal then $I_d = \{(B \rightarrow F) \propto P\}$. Hence, $\langle W, D \rangle \models_s F$ and $\langle W, D \rangle \models_s B$. Notice that neither $\langle W, D \rangle \not\models_c P$ nor $\langle W, D \rangle \not\models_c \neg P$.

Example 4.

$$W = \{P, P \rightarrow B\}$$

$$D = \{(B \rightarrow F) \propto P, (P \rightarrow \neg F) \propto\}$$

For intuition, consider B stands for *Bird*, P stands for *Penguin*, F stands for *Fly*. Any interpretation I such that $I \models P$, $I \models B$ and $I \models \neg F$, satisfies $\langle W, D \rangle$. Notice that if an interpretation I is maximal, $I_d = \{(P \rightarrow \neg F) \propto\}$. Hence, $\langle W, D \rangle \models_s P$; $\langle W, D \rangle \models_s B$ and $\langle W, D \rangle \models_s \neg F$.

Example 5.

$$W = \{R \wedge Q\}$$

$$D = \{(Q \rightarrow P) \propto, (R \rightarrow \neg P) \propto\}$$

For intuition, consider R stands for *Republican*, Q stands for *Quaker*, P stands for *Pacifist*. Any interpretation I such that $I \models R$, $I \models Q$, and $I \models P$, or any interpretation I' such that $I' \models R$, $I' \models Q$, and $I' \models \neg P$ are maximal interpretations for $\langle W, D \rangle$. Notice that $I_d = \{(Q \rightarrow P) \propto\}$ and $I'_d = \{(R \rightarrow \neg P) \propto\}$. Hence, $\langle W, D \rangle \models_c P$ and $\langle W, D \rangle \models_c \neg P$.

Theorem 3 assures that indeed all default theories $\langle W, D \rangle$ are satisfied by a maximal

interpretation as long as W is consistent. The proof is just an adaptation of Lindenbaum theorem that asserts that any consistent set of first-order formulae can be extended to a maximal consistent set.

Theorem 3. Let $\langle W, D \rangle$ be a default theory where W is consistent. Then, there exists a maximal interpretation I that satisfies $\langle W, D \rangle$.

4 Paraconsistent Default Logic

Some authors [10] have argued about the intricacy between nonmonotonicity and paraconsistency. They claim that these two properties are complementary. In every pattern of reasoning where nonmonotonicity arises, paraconsistency is present as well, for in dealing with incomplete, partial and vague information, the rising of conflict and contradiction is inevitable. Hence, many have been the proposals of formalizing nonmonotonic reasoning using paraconsistent logic [11, 12, 13]. In this section we benefit from our approach for default logic presented in section 3 and we generalize it for introducing a paraconsistent nonmonotonic logic, using the multi-valued logic *FOUR* of Belnap [14] as the underlining logic for the semantics. *FOUR* got its name for having four truth values: the classical ones t, f , and two new ones: \perp that intuitively denotes lack of evidence and T that indicates “over” evidence, i.e., evidence for the truth and the falsity of a proposition.

The language is the same as in the maximal default logic but for simplicity sake we focus on the propositional language, using the new connective \propto for representing defaults. A default theory is a pair $\langle W, D \rangle$ of sets of formulae and defaults and all preliminary definitions remain unchanged.

The connectives \wedge, \rightarrow and \neg get the following truth tables:

\wedge	t	f	\perp	T	\rightarrow	t	f	\perp	T	\neg	
t	t	f	\perp	T	t	t	f	\perp	T	t	f
f	f	f	f	f	f	t	t	t	t	f	t
\perp	\perp	f	\perp	f	\perp	t	t	t	t	\perp	\perp
T	T	f	f	T	T	t	f	\perp	T	T	T

A *valuation* is a mapping from the set of propositional letters into $\{t, f, T, \perp\}$. Any valuation is extended to complex formulae in the obvious way. The *designated values* are t and T . That means that a valuation v satisfies a formula φ (denoted as $v \models \varphi$) iff $v(\varphi) = t$ or $v(\varphi) = T$.

The satisfaction of a default is defined as before, $v \models \beta \propto \gamma$ iff $v \models \beta$ and $v \not\models \gamma$. Satisfaction of a default theory also goes as before. Given a default theory $\langle W, D \rangle$ and a valuation v , define

$v_D = \{\beta \propto \gamma \in D ; v \models \beta \propto \gamma\}$. v satisfies a default theory $\langle W, D \rangle$ ($v \models \langle W, D \rangle$) iff v satisfies W ($v \models W$) and v_D is maximal, in the sense that, for all v' such that v' satisfies W and $v_D \subseteq v'_D$, then $v_D = v'_D$. v is called a *maximal valuation* for $\langle W, D \rangle$.

A default theory $\langle W, D \rangle$ *entails* a formula α ($\langle W, D \rangle \models \alpha$) iff for all v such that $v \models \langle W, D \rangle$, $v \models \alpha$.

Example 6.

$$W = \{B, P \rightarrow B\}$$

$$D = \{(B \rightarrow F) \propto P\}$$

For intuition, consider B stands for *Bird*, P stands for *Penguin*, F stands for *Fly*. Any v such that $v \models B$, $v \models F$, and $v \not\models P$, satisfies $\langle W, D \rangle$. Notice that if a valuation v is maximal, $v_D = \{(B \rightarrow F) \propto P\}$. Hence, $\langle W, D \rangle \models F$ and $\langle W, D \rangle \models B$. Notice that neither $\langle W, D \rangle \models P$ nor $\langle W, D \rangle \models \neg P$.

Example 7.

$$W = \{P, P \rightarrow B\}$$

$$D = \{(B \rightarrow F) \propto P, (P \rightarrow \neg F) \propto\}$$

For intuition, consider B stands for *Bird*, P stands for *Penguin*, F stands for *Fly*. Any v such that $v \models P$, $v \models B$ and $v \models \neg F$, satisfies $\langle W, D \rangle$. Notice that if v is maximal, $v_D = \{(P \rightarrow \neg F) \propto\}$. Hence, $\langle W, D \rangle \models P$; $\langle W, D \rangle \models B$ and $\langle W, D \rangle \models \neg F$.

Example 8.

$$W = \{R \wedge Q\}$$

$$D = \{(Q \rightarrow P) \propto, (R \rightarrow \neg P) \propto\}$$

For intuition, consider R stands for *Republican*, Q stands for *Quaker*, P stands for *Pacifist*. Any v such that $v \models R$, $v \models Q$, and $v(P) = T$ is maximal. That is, if v is maximal, then $v_D = \{(Q \rightarrow P) \propto, (R \rightarrow \neg P) \propto\}$. Hence, $\langle W, D \rangle \models P$ and $\langle W, D \rangle \models \neg P$.

As in the logic *FOUR* there are no inconsistent theories, then any default theory has a maximal valuation.

Theorem 2. Let $\langle W, D \rangle$ be a default theory. Then, there exists a maximal valuation v that satisfies $\langle W, D \rangle$.

5 Application and further work

In this paper we presented the maximal default logic that improves Reiter's default logic in many aspects. It is also presented a semantical approach for default reasoning based on maximal sets. This approach revealed to be very profitable and we generalize it to introduce a paraconsistent nonmonotonic logic using the many-valued logic *FOUR* of Belnap as the underlining formalism. Arieli and Avron [13] also developed a paraconsistent nonmonotonic logic based on the logic *FOUR*. A future work should present a through comparison between these logics finding similarities, dissimilarities, advantages and disadvantages of both approaches.

Nonmonotonic reasoning has mainly been studied for application in Artificial Intelligence. We believe, now that the area has been developed to a considerable extent, it may be useful to a wide range of human knowledge, specially in philosophy of science [15]. A theory to be considered scientific must comply with rigid criteria and live up to strict standards. Notwithstanding that, the state of the art of science evolves in a rather chaotic scenario, with the emergency and simultaneous acceptance of incompatible theories. All this scenario of conflicting, incomplete and vague

information forms the ambiance from where nonmonotonic reasoning has emerged. Therefore, we regard that well understood nonmonotonic logics might be of much use for the understanding and normalization of progress in science.

6 References

- [1] M. Pequeno. Maximal default logic. Submitted to the 6th. International Conference on Logic Programming and Nonmonotonic Reasoning – LPNMR’01, Viena, September, 17-19, 2001. (<http://www.lia.ufc.br/~marcel/papers2001/lpnmr2001.html>)
- [2] R. Reiter. A logic for default reasoning. *Artificial Intelligence*, 13 (1980) 81-132.
- [3] D.G. Bobrow & P.J. Hayes (eds). Special issue on non-monotonic logic. *Artificial Intelligence*, 13 (1980)
- [4] S. Hanks & D. McDermott. Nonmonotonic logic and temporal projection. *Artificial Intelligence*, 33 (1987) 379-412.
- [5] D.W. Etherington. Formalizing nonmonotonic reasoning systems. *Artificial Intelligence*, 31 (1987) 41-85.
- [6] P. Besnard and T. Schaub. Possible worlds semantics for default logic. In J. Glasgow and B. Hadley (eds.), *Proceedings of the Canadian Artificial Intelligence Conference*. Morgan Kaufmann Publishers Inc, 1992, 148-155.
- [7] R. A. de Guerreiro and M. A. Casanova. An alternative semantics for default logic. In K. Konolige (ed.), *Third International Workshop on Nonmonotonic Reasoning*, South Lake Tahoe, 1990, 141-157.
- [8] M. Pequeno. Normally canaries fly. In *Proceedings of the Sixth Brazilian Symposium on Artificial Intelligence*, Rio de Janeiro, 1989.
- [9] M. Pequeno. Defeasible reasoning with exceptions first. PhD Thesis, Computing Department, Imperial College, London, 1994.
- [10] T. Pequeno and A. Buchsbaum. The logic of epistemic inconsistency. In *Proceedings Second International conference on principles of Knowledge Representation and Reasoning*, Cambridge, Massachusetts. Morgan Kaufmann Publishers Inc, 1991, 453-460.
- [11] M. Kifer and E. L. Lozinskii. A logic for reasoning with inconsistency. *J. Automated Reasoning* 9 (2) (1992) 179-215.
- [12] G. Priest. Reasoning about truth. *Artificial Intelligence* 39 (1989) 231-244.
- [13] O. Arieli and A. Avron. The value of four values. *Artificial Intelligence* 102 (1998) 97-141.
- [14] N.D. Belnap. A useful four-valued logic. In: G. Epstein, J.M. Dunn (eds), *Modern uses of multiple-valued logic*. Reidel Publishing Company, Boston, 1977, pp. 7-73.
- [15] T. Pequeno, A. Buschsbaum & M. Pequeno. A positive formalization for the notion of pragmatic truth. *Proceedings of the 2001 International Conference on Artificial Intelligence (IC-AI’2001)*, International Workshop Computational Models of Scientific Reasoning and Applications (CMSRA-2001), Special Session on Paraconsistent Logics, Las Vegas, USA, June, 26-28, 2001. This conference.