A Positive Formalization for the Notion of Pragmatic Truth[1]

Tarcisio Pequeno
Laboratório de Inteligência Artificial
Universidade Federal do Ceará
Fortaleza – CE – Brazil

Arthur Buchsbaum
Departamento de Informática e Estatística
Universidade Federal de Santa Catarina
Florianópolis – SC – Brazil

Marcelino Pequeno
Laboratório de Inteligência Artificial
Universidade Federal do Ceará
Fortaleza – CE – Brazil

Abstract

A logic aimed to formalize the concept of pragmatic truth is presented. We start by examining a previous attempt of formalization by da Costa and collaborators, reported in [5], [3] and [2]. However, their formalization works as mere possibility in face of what is known, or assumed. It is pointed out here that not being in conflict with the assumed knowledge is not enough to regard a proposition as a truth of any sort, providing just a necessary condition. A typical picture of the way a scientific theory evolves exhibit alternative hypothesis competing for expanding the theory. In our view, a pragmatic knowledge, at this stage of development of the theory, is one that can be taken as true under all those competing hypothesis. The logic presented here formalizes this process of theory evolution in order to properly express the notion of pragmatic truth as we understand it.

1. Introduction

In [5], an attempt is made to provide a mathematical account for the concept of *pragmatic truth*, further developed into a *logic of pragmatic truth*, in [3] and [2]. As declared by the authors, their formalization is intended to be directly connected to, and is inspired by, the pragmatic conception of truth as introduced by the pragmatists philosophers James, Dewey and Peirce. From the latter, they use to quote the following passage in order to illustrate their motivation: "Consider what effects, that might conceivably have practical bearings, we conceive the object of our conceptions to have. Then, our conception of these effects is the whole of our conception of the object." ([6], pg. 31). They also claim that their formalization captures the idea of a theory *saving the appearances* (as is declared in [2]).

We start by examining the formalization proposed in the mentioned work. This is done in order to show why — in despite of being an interesting attempt and of getting valuable insights on some key issues of the concept of pragmatic truth — their formalization does not really succeed in capturing this notion, nor the related one of *approximation to truth*, or *quasi-truth*, as they occasionally prefer to call it (see [3]). Nevertheless, it can be taken as a good starting point towards the intended formalization, provided that some missing parts in their proposal are conveniently supplied.

[1] Research partially supported by CNPq, project Formalization and Automatization of Reasoning.

Briefly, their notion of pragmatic truth can be described along the following lines. Let T be an accepted theory, expressed in first order predicate logic. Let F be a collection of observed facts (the term "fact" is taken here as in the knowledge basis jargon in artificial intelligence: as ground instances of the predicates of the language L of T). In a domain D of objects, these facts define a collection of partial relations, each one corresponding to a predicate of L. For each partial relation defined by those facts, take a total relation extending it. This will make a structure that can be used to interpret T in D. Now, restrict yourself to the structures so constructed that are models of T. Of course, the facts are supposed to be true in those models. If not, T should be revised (or the facts, who knows?). The mathematical concept of *pragmatic truth,* according to da Costa et al, is then reached. An assumption can be said *pragmatically true* if it is true in any of those structures. This concept is clearly paraconsistent, for it can happen P(a) belonging to such an structure and ¬P(a) to another. The resulting logic, as described by this semantic, resembles Jaskowsky discursive logic. As it is shown in [2], the notion so defined can be identified with possibility in S5 modal logic. Here relies our main point of criticism of this approach. Just being consistent with what is assumed does not turn an assumption into a truth. It is just not prevented of being so, meaning that it remains open as a possibility of truth. At best it could be regard as *pragmatically possible*, a possibility of truth as far as we know at the present time. Along of the present paper we will show that not even so. The concept of *pragmatic possibility* which is defined here as a kind of collateral result is still strictly stronger than this one, not to mention the *pragmatic truth* itself.

In order to reexamine the question of characterizing pragmatically true statements, and to provide a logic formalization to them, we started by considering the *Logic of Epistemic Inconsistency*, LEI, introduced by the authors in [7]. LEI is intended to provide a formalization for the reasoning under conditions of incomplete knowledge, a problem that resembles but is not exactly the same we are considering here. So, it is taken just as a departure point and it has to be conveniently modified in order to express the conditions of the knowledge representation and reasoning in scientific theory, which is our concern here. This is done by the introduction of two kinds of modalities. One called *weak plausibility*, which is also paraconsistent, but that can be shown to be strictly stronger than possibility, and one called *strong plausibility,* which is no longer paraconsistent, but is still revisable, revocable, nonmonotonic. The logical account for this last concept is our proposal for the formalization of *pragmatic truth*.

Since the concept of plausibility plays such a central role here, let us discuss it in some details. In general terms, the concept of plausibility should be emphatically distinguished from notions such as probability, in the sense of "being probable to be true", and should not be confused with mere possibility, not even in the sense of "being possible to be true, in relation to which is already known". This would be too weak to play the role of plausibility. In order to be taken as plausible, an assumption should cope with two kinds of criteria:

1st) It should be sound, in the sense of being consistent with what we agree to take as knowledge. This provides a kind of negative criterion – a conjecture should not be in disaccord with the established theory it tries to extend.

2nd) It should be supported by some kind of positive indication, coming from observation, intuition, analogy, law likeness, simplicity, whatever. In other words, a plausible assumption, or a conjecture, is something to be filtered by means of the creative exercise of elaboration of hypotheses, selected among those alternatives allowed by the theory under development and by experimental observations.

Those considerations on demarcating the notion of plausibility are reflected in the proposed formalization. For instance, the avoidance of the assimilation, or confusion, of plausibility with probability, a very common, although unsound, practice, plays a definite role here. In all acceptable mathematical expression of probability the sum of the probability of a fact and its negation, expressed as its complement, should be 1. This means that if plausibility is defined as something having a probability somewhat close to one, which could seem a reasonable way to express it, one statement and its negation could never be both plausible at the same time. It also entails that if two opposites are equally probable no one of them is really plausible. As it will be seen, in our formalization this does not occur. As we said, this concept, in its weak expression, which is taken as plain plausibility in LEI, is paraconsistent. It may happen a statement and its negation being both plausible when they are both consistent with the accepted facts and each one of them is supported by some hypothesis.

Taken these guidelines into account, we designed a logical system consisting in nonmonotonic rules, which is our mechanism for the introduction of hypotheses, operating over a deductive logic taken as its monotonic basis. This logic, which we call the *logic of appearance*, is a modal logic featuring four modalities: the usual alethic modalities of possibility and necessity and two additional epistemic modalities, the *weak* and *strong plausibilities*. The nonmonotonic apparatus used for the introduction of hypotheses works also as a kind of control mechanism. It provides a test to guarantee that the new hypotheses do not contradict the assumed theory and the known facts. This feature confers dynamics to the developing theory by being to retreat the hypotheses which are in disaccord to new experimental facts introduced in the knowledge field. The new hypotheses so introduced may sum up, in the building of a extended theory, or they may clash with each other forming alternative ramifications of the theory. A proposition true in one of those competing theories corresponds in our logic to the concept of *weak plausibility*. This is our candidate for the notion of *pragmatic possibility*. This concept is still stronger than the *pragmatic truth* of da Costa et al, although being also paraconsistent. In the context of our logic, *pragmatic possibility* can be shown to be strictly stronger than plain possibility. It applies to a smaller set of statements: the ones that, besides possible, are positively sanctioned by at least one existing hypothesis.

Our concept of *pragmatic truth* can now be introduced. A statement is *pragmatically true* if it is true in all competing theories. In other words, if it is so far a consensus. (Being a consensus, a point of convergence, is a property usually required by philosophers to this concept). The authors believe that the concept so formalized really resembles a truth or behaves like a truth in face of all our practical concerns, including our best guesses.

2. The Logic of Appearance

The *Logic of Appearance* is a deductive modal logic aiming to provide a formalization for the concepts of *credulous*, also called *weak*, and *skeptical*, also called *strong*, *plausibility*. It also includes the standard modalities of *possibility* and *necessity*, in order to make possible a formal comparison between them. A language for LA is a first order language, as it is usually defined in standard textbooks [4,8], adopting "$\rightarrow$", "$\neg$", " " and "!" as primitive connectives, and "$\forall$" as the primitive quantifier.

2.1 Notation: From now on, unless declared otherwise, the following conventions are adopted:

- x,y,z represent variables for any language of LA;
- t,u represent terms;
- P,Q,R,S represent formulas of LA;
- Γ,ϑ represent collections of formulas of LA.

2.2 Definition: A *variable is free in* P if it occurs in P out of the scope of "$\forall$". A formula P is said -closed if P has one of the forms Q, Q!, $\neg$R, R $\rightarrow$ S or $\forall$x R, whereon R and S are -closed; otherwise it is said that is free in P. P(x|t) is the formula obtained from P by substituting t for each free occurrence of x.

2.3 Definition: The calculus for LA has the following postulates (axiom schemes and inference rules), whereon, for each inference rule, a varying object[2] is attached:

(→-1) P → (Q → P);

(→-2) (P → Q) → (P → (Q → R)) → (P → R);

(→-3) P, P → Q / Q, whereon there is no attached varying object;

(¬-1) (¬P → Q) → (¬P → ¬Q) → P;

(∀-1) ∀x P → P(x|t), considering the usual restrictions;

(∀-3) P → ∀x P, whereon x is not free in P;

(∀-2) ∀x (P → Q) → (∀x P → ∀x Q);

(∀-4) P / ∀x P, whereon x is the attached varying object;

(-1) P → P;

(-3) P → P, whereon is not free in P;

(-2) (P → Q) → (P → Q);

(-4) ∀x P → ∀x P;

(-5) P / P, whereon is the attached varying object;

(!) P → P!;

(!-1) P! → P, whereon is not free in P;

(!-3) (P! → P)!;

(!-2) P → P!, whereon is not free in P;

(!-3) (P → Q)! → (P! → Q!);

(!-5) (¬P)! → ¬(P!);

(!-6) ∀x (P!) → (∀x P)!.

2.4 Definition: As usual, a consequence relation "$\vdash_{LA}$" is defined relating collections of formulas in LA to formulas of LA. Beyond that, it is defined "$\vdash^{V}_{LA}$", whereon *V* is a collection of varying objects:

- a deduction D in LA *depends on* a collection *V* (of varying objects) if *V* contains the collection of varying objects o of all applications of rules in D having a hypothesis in which o is free such that there is a formula, justified as a premise in D, whereon o is free too, relevant to this hypothesis in D.
- P *is a consequence of* Γ in LA *depending on V* if there is a deduction of P from Γ in LA depending on *V*; it is noted by Γ $\vdash^{V}_{LA}$ P.

2.5 Notation: The following abbreviations are adopted:

- P ∧ Q ≡ ¬(P → ¬Q);
- P ∨ Q ≡ ¬P → Q;
- P ↔ Q ≡ (P → Q) ∧ (Q → P);
- ∃x P ≡ ¬∀x ¬P;
- ◊P ≡ ¬ (¬P);
- P? ≡ ¬(P!).

As it is usual in modal logic, the signs " " and "◊" stand for *necessity* and *possibility* respectively, while the signs "!" and "?" stand respectively for *skeptical* and *credulous plausibility*.

2.6 Theorem: All the signs "→", "∧", "∨", "¬", "↔", "∀" and "∃" behave in LA like in open classical logic[3]. Below it is formulated the deduction theorem:

- if Γ ∪ {P} $\vdash^{V}_{LA}$ Q and no varying object in V is free in P, then Γ $\vdash^{V}_{LA}$ P → Q.

2.7 Theorem: The following propositions show the interrelationship among necessity, skeptical plausibility, credulous plausibility and possibility:

- $\vdash_{LA}$ P → P!;
- P! / P is not a valid rule in LA;
- $\vdash_{LA}$ P! → P?;
- P? / P! is not a valid rule in LA;
- $\vdash_{LA}$ P? → ◊P;
- ◊P / P? is not a valid rule in LA.

2.8 Definition: Γ is said LA-*trivial* if Γ $\vdash_{LA}$ P, for each formula P in LA.

This completes the exposition of the monotonic part of our system. It provides a logical environment for the analysis of the concepts of *pragmatic truth* and *pragmatic possibility*, as intended by the authors. However, in order to make this analysis really comprehensive, a more general setting able to express the dynamics of development of scientific theories, at least in its most basic components, must be introduced. This will correspond to the nonmonotonic part of our system, to be presented in the next section.

---

[2] A variable in LA or the symbol " ".

[3] As it is presented, for example, in [4] or [8]; in [1] general concepts about open calculi, varying objects and deduction theorems are analyzed.

3. A Logic for Pragmatic Truth

In this section a nonmonotonic extension for the *Logic of Appearance* is defined. It provides a key issue in our strategy of formalization for the concept of *pragmatic truth*, for this could never been done without considering the process of adding hypotheses to scientific theories, the mechanism that enable theories to grow and develop. Concepts concerning scientific reasoning, in general, can be appreciated only from this perspective. So, the logical system resulting from the extension of LA by the nonmonotonic mechanism to be now introduced makes what we call the "*Logic of Pragmatic Truth*", shortly LPT.

3.1 Notation: From now on, the following conventions are adopted:

- L is a language for LA;
- P, Q are formulas of L;
- ϑ is a collection of formulas of L.

3.2 Definition: A *generalization* (*in* L) is an expression of the form P —(Q; if P,Q are closed formulas, it is said that this expression is a *closed generalization* (*in* L). An LPT-*basis* (*in* L) is a pair $\Delta = \langle W,G \rangle$, whereon W is a collection of formulas (of L), and G is a collection of generalizations (in L). An *instance of a generalization* P —(Q (in L) is an expression P' —(Q', whereon P',Q' are closed consistent instances[4] of P,Q (in L).

3.3 Definition: If G is a collection of generalizations, then it is specified:

- Rest(G)[5] ≡ {Q / there exists P such that "P —(Q" belongs to G};
- Conj(G)[6] ≡ {P / there exists Q such that "P —(Q" belongs to G}.

3.4 Definition: If $\vartheta = \{P_1,\ldots,P_n\}$, then:

- $\vartheta \equiv P_1 \wedge \ldots \wedge P_n$;
- $\vartheta \equiv P_1 \vee \ldots \vee P_n$.

3.5 Notation: From now on, $\Delta = \langle W,G \rangle$ is a LPT-basis in L.

3.6 Definition: $\Gamma_\Delta(\vartheta)$ and $\overline{\Gamma}_\Delta(\vartheta)$ are respectively the least collections of formulas of L and of sets of instances of generalizations of G in L satisfying the following conditions:

- $W \subseteq \Gamma_\Delta(\vartheta)$;
- if $\Gamma_\Delta(\vartheta) \vdash_{LA} P$, then $P \in \Gamma_\Delta(\vartheta)$;
- if G' is a finite collection of instances of generalizations of G in L such that (Rest(G'))? $\notin \vartheta$, then, for each subset G'' of G', if $\vartheta \cup \{$(Conj(G''))?$\}$ is not LA-trivial, then (Conj(G''))? $\in \Gamma_\Delta(\vartheta)$ and $G'' \in \overline{\Gamma}_\Delta(\vartheta)$.

3.7 Definition: E is an *extension in* Δ if $\Gamma_\Delta(E) = E$.

3.8 Definition: A generalization is said *triggered in* Δ if it belongs to $\tau\overline{\Gamma}_\Delta(E)$, for each extension E in Δ. A collection G' of triggered generalizations in Δ is said *compatible in* Δ if there exists an extension E of Δ such that, for each finite subset G'' of G', $G'' \in \overline{\Gamma}_\Delta(E)$. G' is said *maximal compatible in* Δ if it is compatible in Δ and, for each G'' compatible in Δ, if $G' \subseteq G''$, then G' = G''.

3.9 Definition: A generalization is said *strongly triggered in* Δ if it belongs to each maximal compatible collection of generalizations in Δ.

3.10 Definition: ♥(Δ) is the least collection of formulas of L satisfying the following conditions:

- $W \subseteq$ ♥(Δ);
- if ♥(Δ) $\vdash_{LA}$ P, then P $\in$ ♥(Δ);
- if P is a sentence of L and $W \cup \{\lozenge P\}$ is not LA-trivial, then $\lozenge P \in$ ♥(Δ);
- if G' is a finite compatible collection of triggered generalizations in Δ, then (Conj(G'))? $\in$ ♥(Δ);
- if P —(Q is strongly triggered in Δ, then P! $\in$ ♥(Δ);
- if P —(Q is strongly triggered in Δ and $W \cup \{$ P$\}$ is not LA-trivial, then P $\in$ ♥(Δ).

3.11 Definition: $\Delta \vdash_{LPT}$ P ≡ P $\in$ ♥(Δ).

3.12 Scholium: The four modalities maintain in LPT a relationship analogous to the one already expressed for LA in theorem **2.7**.

[4] That is, variables occurring both in P and Q are replaced by the same closed terms.

[5] "Rest(G)" is read "restrictions of G".

[6] "Conj(G)" is read "conjectures of G".

4. Conclusions

Of course, in face of a proposed mathematization for a given concept, such as the one just presented, apart of the details concerning its technical execution, of its strictly logical or mathematical aspects, which we can call *internal properties* of the formal system, a discussion remains on whether the essential features of the concept under consideration were really captured by the proposed model. This is the case of almost all logical formalizations of common or philosophical concepts, such as the logical connectives, implication, or modalities, for instance. It is no exception here. The affairs are even made worst by the fact that the concept on focus is itself not a very clear one in the first place. A concept, which usually in the philosophical literature has been more suggested than really defined, given room to many different interpretations and formulations, depending on the interest of the one using it. As a matter of fact, one of the points in formalizing a concept is exactly to make precise its reference and to stabilize its interpretation from that point on. As a rule, a great amount of discussion is required until a consensus can be reached and convention of use adopted, if ever. So, the authors have no illusions of been attaining a consensus here and now by offering the ultimate mathematical interpretation for the pragmatic truth concept.

However, we are in the position of defending our proposal by supporting it with some arguments and by justifying our choices among some alternatives. We start by briefly recalling its main features.

We started by stating a logic aiming to express the notion of plausibility in two variants, a *credulous*, or *weak*, and a *skeptical*, or *strong* one, through our *logic of appearance*. Besides its axiomatization, a semantics, that was not included for lack of space here, has been also developed for it. This semantics could help the intuition about this logic. It is based on the possible world framework, with the alethic modalities defined as in S5, but also considering a subset of the possible worlds, which we call *plausible worlds*. Thus, a *weak plausibility* is a proposition true in one of these worlds, while a *strong plausibility* is true in all of them. Of course a completeness proof was provided (this work remains unpublished). This logic has some interest and uses on its own. In order to play the role we need here, as the formal basis for the analysis of pragmatic truth and possibility, a second major step, concerning the process of introduction of hypotheses in scientific theories, is required.

A typical picture of the way a scientific theory evolves consists in some alternative hypothesis competing for the right of providing extensions for the theory. This process is represented here by the use of nonmonotonic rules, able to introduce and to retreat hypothesis, as the case may be. This is the way through which the epistemic modalities related to plausibility appear at the deductive level. This is the mechanism by which, in this logic, plausibility is made into *pragmatism.* A pragmatic knowledge, a knowledge that really *saves the appearances* at a certain stage of development of a scientific theory, is then one that can be taken as true under all those competing hypothesis; in other words, one that satisfies all those rules, thus belonging in all the plausible worlds sanctioned by each of them.

Our justification of being faithful to this concept, as proposed by the pragmatists, is that this construction really represents what can be taken as true in face of all our practical concerns, reflected in our knowledge so far, including our best guesses.

A typical example frequently invoked to express the usefulness of the idea of a truth being taken pragmatically is the adoption, for the sake of simplicity, of newtonian mechanics for applications were the velocities involved are relatively low, as is the case in sending rockets to the moon. In our picture, relativity and classical mechanics would be considered as competing hypotheses for this effect. In the region under concern, all the predictions of both theories agree at the level of precision of our instruments, or of our particular needs, enabling the use of any of these theories, and, of course, the adoption of the simpler one, as pragmatically true.

References

[1] Buchsbaum, A. R. V. and Pequeno, T. H. C., *A General Treatment for the Deduction Theorem in Open Calculi*, Logique et Analyse, Vol. 157: 9-29, 1997.

[2] da Costa, N. C. A., Bueno, O. and French, S., *The Logic of Pragmatic Truth*, Journal of Philosophical Logic, 27: 603-620, 1998.

[3] da Costa, N. C. A., Chuaqui, R. and Bueno, O., *The Logic of Quasi-Truth*, Notas de la Sociedad de Matemática de Chile, Vol. XV: 7-26, 1996.

[4] Mendelson, E., *Introduction to Mathematical Logic*, D. Van Nostrand, 1979.

[5] Mikenberg, I., da Costa, N. C. A., and Chuaqui, R., *Pragmatic Truth and Approximation to Truth*, The Journal of Symbolic Logic 51: 201-221, 1986.

[6] Peirce, C. S., *Collected Works*, Ed. By J. Buchler, New York, Dover, 1965.

[7] Pequeno, T. H. C. and Buchsbaum, A. R. V., *The Logic of Epistemic Inconsistency*, Principles of Knowledge Representation and Reasoning: Proceedings of the Second International Conference: 453-460, 1991.

[8] Shoenfield, J. R., *Mathematical Logic*, Addison-Wesley, 1967.