The Logic of Intra-Theoretical Scientific Reasoning

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1. Introduction

Our main topic here is what can be called *intra-theoretical scientific inference*. By this term we mean the inferences performed inside scientific theories aimed to go from the theory's basic principles to the derived ones. This kind of inferences plays an important role, for example, in the explanation of scientific laws and singular facts as well as in the prediction of non-observed facts.

The traditional view concerning intra-theoretical scientific inference states that scientific arguments are of two types: deductive and inductive/probabilistic. According to this view, all inferences performed inside scientific theories that cannot be properly represented within deductive logic can be so represented within an inductive-probabilistic framework. In other words, all 'informal' intra-theoretical scientific arguments can in principle be retrospectively justified as acceptable arguments by being fitted either into a deductive or into a probabilistic conceptual frame.

This classification of inferences as either deductive or probabilistic is closely connected to Rudolf Carnap's philosophy of induction and probability.[1] As used by Carnap, the terms 'probability' and 'induction' embody two reductions. First of all, since Carnap's theory of induction aims to account for the non-deductive inferences that have a kind of 'logical rectitude', non-deductive is reduced to inductive. Second, since Carnap's solution to the question of what this logical rectitude is about consists of saying that it makes the conclusions obtained from a set of true premises not true but *probable*, inductive is reduced to probabilistic. It is worthy to note that, although Carnap correctly identified three different notions of probability – the classificatory, the comparative and the quantitative probability – probability as comprised in this reduction should be understood as a quantitative notion.

This deductive/inductive-probabilistic view of intra-theoretical scientific inferences was put forward in its most precise form by Carl Hempel's models of scientific explanation. Following Carnap's view on inferences, Hempel proposed his probabilistic-based Inductive-Statistical (I-S) model of scientific explanation to represent the *non-deductive* intra-theoretical scientific inferences. This model however was unable to solve satisfactorily the so-called problem of *inductive ambiguities*. As a consequence of this, it revealed the difficulties in representing non-deductive inferences through a numerical-probabilistic approach and, more generally, evidenced the untenability of the deductive-inductive/probabilistic view of intra-theoretical scientific inferences.

Instead of using a quantitative concept as done by the deductive-inductive/probabilistic view, we propose in this essay to represent the non-deductive intra-theoretical scientific inferences through the *qualitative* concept of *plausibility*. By presenting a formal logical system of plausibility, we intend to show how the use of such concept has some interesting advantages over the traditional probabilistic approach. First, it allows a real nonmonotonic inferential mechanism able to capture the non-deductive character of the kind of scientific inference stressed here. Second, since it makes possible the representation of priorities among

[1] See Carnap (1950).

scientific laws and allows for a paraconsistent inferential mechanism, it gives a real solution to the problem of inductive ambiguities.

The structure of the paper is as follows. First of all we will introduce Hempel's I-S model and show the problems with the probabilistic account of non-deductive intra-theoretical scientific inferences. This will be done in the Sections 2 and 3. After that we will present a logical system that formalizes the concept of plausibility and show how it can be used to represent the intra-theoretical scientific inferences and solve the problems presented by Hempel's approach. This will be done in Sections 4 and 5. In Section 6 we will expose some conclusive remarks.

2. Hempel's I-S Model

According to most historiographers of philosophy, the history of philosophical analysis of scientific explanation started with the publication of 'Studies in the Logic of Explanation' in 1948 by Carl Hempel and Paul Oppenheim. In this work, Hempel and Oppenheim propose their deductive-nomological (D-N) model of scientific explanation where scientific explanations are considered as being deductive arguments that contain essentially at least one general law in the premises. Later in 1962, Hempel presented his inductive-statistical (I-S) model by which he proposed to analyze the statistical scientific explanations that clearly could not be fitted into the D-N model. [2]

Because of his emphasis on the idea that explanations are arguments and his commitment to Carnap's theory of inference, Hempel's models perfectly exemplify the deductive-inductive/probabilistic view of intra-theoretical scientific inferences. The D-N model does not pose any problem to the deductive-inductive/probabilistic view. On the other hand however, in its attempt to formalize non-deductive arguments through a quantitative notion, the I-S model presents some difficulties that threat the tenability of this view.

The general schema of non-deductive scientific explanations according to Hempel's I-S model is the following:

$$\frac{\begin{array}{l}P(G, F) = r\\ Fb\end{array}}{Gb}\quad [r]$$

Here the first premise is a statistical law asserting that the relative frequency of Gs among Fs is *r*, *r* being close to 1. The second premise stands for *b* having the property *F*, and the expression '[r]' next to the double line represents the degree of inductive probability conferred on the conclusion by the premises. Since the law represented by the first premise is not a universal but a statistical law, the argument above is inductive (in Carnap's sense) rather than deductive.

If we ask, for instance, why John Jones (to use Hempel's main example) recovered quickly from a streptococcus infection we would have the following argument as the answer:

(1) $$\frac{\begin{array}{l}P(G, F\wedge H) = r\\ Fb \wedge Hb\end{array}}{Gb}\quad [r]$$

where F stands for having a streptococcus infection, H for administration of penicillin, G for quick recovery, b is John Jones, and r is a number close to 1. Given that penicillin was

[2] The 1948 article is reprinted in Hempel (1965). The I-S model appeared for the first time in Hempel (1962) and is explained in detail in Hempel (1965).

administrated in John Jones case (Hb) and that most (but not all) streptococcus infections clear up quickly when treated with penicillin ($P(G,F \wedge H) = r$), the argument above constitutes the explanation for John Jones's quick recovery.

However, it is known that certain strains of streptococcus bacilli are resistant to penicillin. If it turns out that John Jones is infected with such a strain of bacilli, then the probability of his quick recovery after treatment of penicillin is low. In that case, we could set up the following inductive argument:

(2) $P(G, F \wedge H \wedge J) = r'$ or, equivalently, $P(\neg G, F \wedge H \wedge J) = 1 - r'$

$$\frac{\begin{array}{l} P(G, F \wedge H \wedge J) = r' \\ Fb \wedge Hb \wedge Jb \end{array}}{Gb} \quad [r'] \qquad \text{or, equivalently,} \qquad \frac{\begin{array}{l} P(\neg G, F \wedge H \wedge J) = 1 - r' \\ Fb \wedge Hb \wedge Jb \end{array}}{\neg Gb} \quad [1 - r']$$

where J stands for the penicillin-resistant character of the streptococcus infection and r' is a number close to zero (consequently 1 – r' is a number close to 1.)

This situation exemplifies what Hempel calls the problem of explanatory ambiguities or *inductive ambiguities*. In the case of John Jones's penicillin-resistant infection, we have two inductive arguments where the premises of each argument are logically compatible and the conclusion is the same. Nevertheless, in one argument the conclusion is strongly supported by the premises, whereas in the other the premises strongly undermine the same conclusion.

In order to try to solve this sort of problem, Hempel proposed his *requirement of maximal specificity* (RMS). It can be explained as follows. Let *s* be the conjunction of the premises of the argument and *k* the conjunction of all statements accepted at the given time (called knowledge situation). Then, according to Hempel, "to be rationally acceptable" in that knowledge situation the explanation must meet the following condition: If $s \wedge k$ implies that *b* belongs to a class F_1, and that F_1 is a subclass of F, then $s \wedge k$ must also imply a statement specifying the statistical probability of G in F_1, say

$P(G, F_1) = r'$

Here, r' must equal r unless the probability statement just cited is a theorem of mathematical probability theory.[3]

The RMS intends basically to prevent that the property or class F to be used in the explanation of Gb has a subclass whose relative frequency of Gs is different from P(G,F). In order to explain Gb through Fb and a statistical law such as $P(G, F) = 0.9$, we need to be sure that, for all sets $F_1 \in F$ such that F_1b, the relative frequency of Gs among F_1s is the same as that among Fs, that is to say, $P(G, F_1) = 0.9$. In other words, what the RMS states is that, in order to be used in an explanation, the class F must be an *homogeneous* one with respect to G.

3. The Problems with Hempel's Approach

The RMS was proposed of course because of I-S model's inability to solve the problem of ambiguities. Since it allows the appearance of ambiguities and gives no adequate treatment for them, without RMS the I-S model is simply useless as a model of intra-theoretical scientific inferences. But we can wonder: Is the situation different with the RMS?

First of all, in its new version the I-S model allows us to classify arguments as authentic scientific inferences able to be used for explaining or for predicting singular facts only if they satisfy the RMS. It is not difficult to see that this restriction is too strong to be satisfied in

[3] Hempel (1965), pg. 400. For the sake of uniformity we are not using here Hempel's original notation.

practical circumstances. Suppose that we *know* that most streptococcus infections clear up quickly when treated with penicillin, but we *do not* know whether this statistical law is applicable to all kinds of streptococcus bacillus taken separately (that is, we do not know if the class in question is a homogeneous one). Because of this incompleteness of our knowledge, we are not entitled to use argument (1) to explain (or predict) the fact that John Jones had (or will have) a quick recovery. Since when making scientific prediction, for example, we have nothing but imprecise and incomplete knowledge, the degree of knowledge required by the RMS is clearly incompatible with actual scientific practice. In other words, the I-S model imposes a restriction that, due to the very nature of the circumstances in which we perform intra-theoretical scientific inferences, we are not able to satisfy.[4]

Secondly, the only cases that the RMS succeeds in solving are those that involve class specificity. In other words, the only kind of ambiguity that the RMS prevents consists of that that comes from a conflict arising *inside* of a certain class with one of its subclasses. Suppose that John Jones has contracted HIV. As such, the probability of his quick recovery (from any kind of infection) will be low. But given that he took penicillin and that most streptococcus infections clear up quickly when treated with penicillin, we will still have the conclusion that he will recover quickly. Thus an ambiguity will arise from this situation. However, as the class of HIV infected people who have an infection does not belong to the class of individuals having a streptococcus infection which was treated with penicillin (and nor vice-versa), the RMS will not be able to solve the conflict. To sum up, when the ambiguity comes from two classes such that no one of them belongs to the other one, the RMS will fail in his task of preventing ambiguities.

Thirdly, sometimes the policy of preventing all kinds of contradictions may not be the best one. Suppose that the antibiotic that John Jones used in his treatment belongs to a recently developed kind of antibiotic that its creators guarantee to cure even the known penicillin-resistant infection. The initial statistics showed a 90% of successful cases. Even though this result cannot be considered as definitive (due to the always-small number of cases considered in initial tests), it must be taken into account. Now, given argument (2), the same contradiction will arise. But here we do not know yet which of the two ‘laws’ has priority over the other. Maybe the penicillin-resistant bacillus will prove to be resistant even to the new antibiotic or maybe not. Anyway, if we reject the contradiction as the I-S model does and do not allow the use of these inferences, we will loss a possibly relevant part of the total set of information that could be useful or even necessary for other inferences.

4. The Logic of Intra-Theoretical Scientific Reasoning

Compared to the traditional probabilistic-statistical view of non-deductive intra-theoretical scientific inferences, our proposal’s main shift can be summarized as follows. First and most importantly, we are proposing to represent the non-deductive intra-theoretical scientific reasoning through not a quantitative notion, but through the qualitative concept of *plausibility*. Secondly, with the help of this concept we provide a nonmonotonic inferential mechanism through which non-deductive scientific inferences can be represented. Thirdly, in order to prevent the appearance of ambiguities we provide in our formalism a mechanism by which *exceptions* of laws can be represented. This mechanism has mainly two advantages over Hempel’s RMS: it can prevent the class specificity ambiguities without rejecting both arguments (as Hempel’s does) and can also treat properly those cases of ambiguity that do not involve class specificity. Finally, in order to consider the cases where the ambiguity is

[4] One can object that scientific theories actually stipulate ideal universal generalizations that need not necessarily be connected to pragmatic-epistemic considerations. To answer to this objection it suffices to remember that in order to predict or explain singular facts we need the so-called auxiliary hypotheses or initial conditions which do depend on epistemological aspects

due to the very nature of the knowledge to be formalized and, as such, cannot be prevented, we supply a *paraconsistent* mechanism by which certain types of ambiguities can be tolerated without trivializing the set of premises. Consequently, even in the presence of contradictions we can make use of all information contained in the set of premises and keep reasoning without concluding everything from it.

The logical system that we propose to use to formalize the intra-theoretical scientific inferences is a modified version of the logics introduced in Pequeno & Buchsbaum (1991) and further developed in various others works.[5] This system consists basically of two different logics connected to each other in a very important way.

First of all, there is a *nonmonotonic logic* (called Inconsistent Default Logic or IDL) that allows for the performance of nonmonotonic inferences. The conclusions of these nonmonotonic inferences can be marked with the modal operator of *plausibility* **?**, where $\alpha?$ means 'α is plausible.' In this way, differently from the traditional nonmonotonic logics, IDL is able to distinguish refutable formulae obtained though nonmonotonic inferences from non-refutable ones.

Nonmonotonic inferences can be performed with the help of the 'unless' connector $\not\subset$. $\alpha \not\subset \beta$, or 'α unless β', means that the formula α can be used in any inferential chain unless β is one of the conclusions obtained from the set of formulae in question. The nonmonotonicity comes from the fact that, if after having concluded α from $\alpha \not\subset \beta$ some modification in the set of formulae makes β a valid conclusion, we are not entitled to use $\alpha \not\subset \beta$ to infer α any longer.

The formula $\alpha \not\subset \beta$ can be thought of as representing a kind of typicality where α is the fact that typically occurs and β is the exception to this occurrence. If we are to represent non-universal scientific laws such as statistical ones, for example, this non-universality could be represented by the following schema:

$$(\alpha \rightarrow \beta?) \not\subset \varphi$$

This formula states that $\beta?$ can be inferred from α unless the exception, φ, holds. The symbol ? in this case indicates that β is not a certain conclusion, but a plausible one, for if later φ becomes a valid conclusion, $\beta?$ cannot be inferred any longer.

The utility of **?**, however, is not limited to a mere monotonic-nonmonotonic distinguisher. It actually links the two notions of nonmonotonicity and *paraconsistency* by providing a way of distinguishing those formulae to which the non-contradiction principle can be applied from those to which it cannot.

In virtue of the domains in which the nonmonotonicity is required, namely the ones where the knowledge to be represented is incomplete an imprecise, the appearance of contradictions is practically an inevitable phenomenon. If we do like it is suggested here and distinguish certain formulae from refutable ones, we will have basically two kinds of contradictions or ambiguities: strong contradictions such as $\alpha \wedge \neg\alpha$, and weak contradictions such as $\alpha? \wedge (\neg\alpha)?$. While it is reasonable not to allow the first kind of contradiction, we can conceptually admit the truthfulness of sentences such as $\alpha? \wedge (\neg\alpha)?$. But this tolerance with respect to weak contradictions is much more than a mere conceptual possibility. Since that one of the parts of the weak contradiction ($\alpha?$ or $\neg\alpha?$) can be defeated later, it is actually extremely desirable to keep reasoning even with a so-called contradiction. But since we do not want to have every sentence inferred from such kind of contradiction, we need something like a paraconsistent mechanism through which we could be able to reason in the presence of

[5] For a good exposition of Pequeno's logics of plausibility see Martins (1998).

contradictions without trivializing the theory. This mechanism is the second component of our logical system, called *Logic of Epistemic Inconsistency* or, in short, LEI.

The logic LEI is the *monotonic* and *paraconsistent* component that makes the reasoning about the formulae obtained through the nonmonotonic logic IDL possible. It originally was presented through an axiomatic and semantic formulation having its soundness and completeness proved. Below it is showed some important axioms of LEI concerning the plausibility operator **?**: (Latin letters represent exclusively ?-free formulae)

$(\alpha \rightarrow B) \rightarrow ((\alpha \rightarrow \neg B) \rightarrow \neg\alpha)$
$\alpha \rightarrow \alpha?$
$\alpha?? \rightarrow \alpha?$
$(\alpha \rightarrow \beta)? \rightarrow (\alpha? \rightarrow \beta?)$
$(\neg\alpha)? \leftrightarrow \neg(\alpha?)$

The first axiom schema, which is a weaker version of the *reduction ad absurdum* axiom, is the key of LEI's paraconsistency. In classical logic, it is the *reduction ad absurdum* axiom what makes possible that from a contradiction we deduce everything. Here however, since $(\alpha \rightarrow B) \rightarrow ((\alpha \rightarrow \neg B) \rightarrow \neg\alpha)$ can be used only if B is a ?-free formula, it is not possible to make the *reduction ad absurdum* from weak contradictions. In other words, from $\alpha? \wedge (\neg\alpha)?$ it does not follow the trivialization of the theory. Concerning strong contradictions, however, the *reduction ad absurdum* axiom can be used exactly in the same way as in classical logic. Thus, for plausible formulae LEI behaves paraconsistently, but for ?-free formulae it behaves classically.[6]

LEI's language does not include the operator $\not\subset$. It consists basically of propositional logic's language in addition to the operator ?. In its turn, IDL's formulae $\alpha \not\subset \beta$ are such that α and β belong both to LEI's language.

With the help of the monotonic inferential relation $\Box_?$ provided by LEI we can formally define the nonmonotonic inferential relation of IDL. Instead of being a relation between a set of formulae and a formula, IDL's inferential relation has as its parameters a formula and what is called an IDL-theory. An IDL-theory is a pair <W,D> where W is a set of formulae belonging to LEI's language and D is a set of closed $\not\subset$-formulae. The set of all (nonmonotonic) conclusions obtained through an IDL-theory is called the *extension* of this theory. Let <W, D> be an IDL-theory and S be any set of formulae. $\Gamma(S)$ is the smallest set that satisfy the following conditions:

(i) $W \subseteq \Gamma(S)$;
(ii) If $\Gamma(S) \Box_? \phi$, then $\phi \in \Gamma(S)$;
(iii) If $\alpha \not\subset \beta \in D$ and $\beta \notin S$, then $\alpha \in \Gamma(S)$

A set of formulae E is an extension of <W,D> iff $\Gamma(E)=E$.[7]

An equivalent definition of the notion of IDL extension can be done as follows. Given the IDL-theory <W,D>, consider the sequence of sets of formulae S_0, S_1, S_2, … and S such that $S_0 = W$, $S = \Box_{i=0}^{\Box} S_i$, and $S_{i+1} = S_i \cup \{\alpha\text{: } \alpha \not\subset \beta \in D \text{ and } \beta \notin Th_?(S)\}$. The extension of <W,D> is the set of formulae E such that $E = Th_?(S)$, where $Th_?(S)$ is the set of all formulae inferred from A thought LEI's inferential relation $\Box_?$.

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[6] Actually, in LEI all principles of classical logic are respected by ?-free formulae.
[7] Despite the terminology used here, in some cases an IDL-theory can have more than one extension. Thus, properly speaking it is not correct to speak about *the* set of conclusions of an IDL-theory, or even about the inferential relation of IDL as a relation between a formula and an IDL-theory. We do find however that the use of this terms is more adequate for a summarised presentation like this

5. Inductive Ambiguities: The Solution

As Hempel's explanatory ambiguities show, the appearance of ambiguities is an inevitable phenomenon when we deal with non-deductive inferences.[8] Surprisingly enough, all cases of inductive ambiguities identified by Hempel are not due to this suggested connection between ambiguity and induction, but actually to the incapacity of his probabilistic approach to represent properly the situations in question. In this section we will show how the examples that Hempel uses to expose the problem of inductive ambiguity can be easily solved by the approach that we are proposing here.

Consider again John Jones's example. The situation exposed in section 2 can be formalized in our logic as follows:

(3) $((Fx \wedge Hx) \rightarrow Gx?) \not\subset (\neg Gx \vee Jx)$
(4) $Fb \wedge Hb$

Here (3) is a formula schema that says that if someone has a streptococcus infection and was treated with penicillin, then it is plausible that it will have a quick recovery unless it is verified that the quick recovery will not be the case or that John Jones's streptococcus is a penicillin resistant one.[9] (4) states that John Jones has a streptococcus infection and that he took penicillin. Given $W = \{Fb \wedge Hb\}$ and $D = \{((Fb \wedge Hb) \rightarrow Gb?) \not\subset (\neg Gb \vee Jb)\}$ as the IDL-theory, we have that the extension of $<W,D>$ is $E = Th_?(\{Fb \wedge Hb, (Fb \wedge Hb) \rightarrow Gb?\})$. By using *modus ponens* together with the two members of the set E we can easily see that $Gb? \in E$.

Suppose now that we got the new information that John Jones's streptococcus is a penicillin resistant one. We represent this by the following formula:

(4') $Fb \wedge Hb \wedge Jb$

Like in Hempel's formalism, if someone is infected with a penicillin-resistant bacillus, it is not plausible that he will have a quick recovery after the treatment of penicillin (unless we know that he will recover quickly). This can be represented by the following schema of formula:

(5) $((Fx \wedge Hx \wedge Jx) \rightarrow \neg Gx?) \not\subset Gx$

Given $W' = \{Fb \wedge Hb \wedge Jb\}$ and $D' = D \cup \{(Fb \wedge Hb \wedge Jb) \rightarrow \neg Gb?) \not\subset Gb\}$ as our new IDL-theory, we have that the extension of $<W',D'>$ is $E' = Th_?(\{ Fb \wedge Hb \wedge Jb, (Fb \wedge Hb \wedge Jb) \rightarrow \neg Gb?\}$. By the same procedure we can conclude that $\neg Gb? \in E'$.

Since in Hempel's approach there is no connection between laws (1) and (2), the conclusion of $\neg Gb$ has no effect on the old conclusion Gb. Here however it is being represented the priority that we know law (5) must have over law (3). The clauses Jx in the $\not\subset$-right side of (3) and Jx in the $\rightarrow$-left side of (5) taken together mean that if (5) can be used for inferring, for example $\neg Gb?$, (3) cannot be used for inferring Gb?. So, if after using law (3) we get new information that enable us to use law (5), since in the light of the new state of knowledge law (3)'s utilization is not possible, we have to give up the previous conclusion got from this law.

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[8] Before presenting his I-S model and the problem of explanatory ambiguity, Hempel was aware of the connection between inductive reasoning and inconsistencies. See his "inductive Inconsistencies", reprinted as section 2 of Hempel (1965).

[9] In the examples given here we are using the same formulas to represent both cases of explanation and prediction. Since the logical language does not have future operators, the setting of the context (whether explanation or prediction) is being made in the explication of the formula.

Turning back to John Jones example, since $(\neg Gb \vee Jb) \in E'$, we are no longer entitled to infer $(Fb \wedge Hb) \rightarrow Gb?$ from (3) and, consequently, cannot infer Gb? any more. The only plausible fact that we can conclude from (3) and (5) is $\neg Gb?$. As such, in contrast to Hempel's approach, we do not have the undesirable consequence that it is plausible (or in Hempel's approach, high probable) that John will quickly recover and that it is plausible that he will not.

As it was said, in this specific case we know that law (5) has a kind of priority over law (3), in the sense that if (5) holds, (3) does not hold. Like in Section 3, suppose now that the antibiotic that John Jones used in his treatment belongs to a recently developed kind of antibiotic that its creators guarantee to cure even the known penicillin-resistant infection. The initial statistics showed a 90% of successful cases but due to the always-small number of initial cases, this result cannot be considered as definitive. Even so, we can set up the following tentative law:

(6) $((Fx \wedge H'x) \rightarrow Gx?) \not\subset \neg Gx$,

where H' stands for administration of the new kind of antibiotic. To complete this new example we have the two following formulae:

(4'') $Fb \wedge H'b \wedge Jb$

(7) $\forall x(H'x \rightarrow Hx)$

Given $W'' = \{ Fb \wedge H'b \wedge Jb , \forall x(H'x \rightarrow Hx)\}$ and $D'' = \{((Fb \wedge Hb \wedge Jb) \rightarrow \neg Gb?) \not\subset Gb , ((Fb \wedge H'b) \rightarrow Gb?) \not\subset \neg Gb,\}$ (laws (5) and (6)) as our new IDL-theory, we have that the extension of $<W'',D''>$ is $E'' = Th_?(\{ Fb \wedge H'b \wedge Jb , ((Fb \wedge Hb \wedge Jb) \rightarrow \neg Gb?) \not\subset Gb , ((Fb \wedge H'b) \rightarrow Gb?) \not\subset \neg Gb\}$. Clearly $Gb? , \neg Gb? \in E''$.

In this case, we do not know which of the two 'laws' has priority over the other. Maybe the penicillin-resistant bacillus will prove to be resistant even to the new antibiotic or maybe not. Instead of rejecting both conclusions, as I-S model with its RMS would do, we defend that a better solution is to keep reasoning even in the presence of such ambiguity, but without allowing that we deduce everything from it. Formally this is possible because of the restriction imposed by the already shown LEI's axiom of non-contradiction. If a modification that resolves the conflict is made in the set of facts (a change in (5) representing the definitive success of the new kind of penicillin, for example) the IDL's nonmonotonic inferential mechanism will update the extension and exclude one of the two contradictory conclusions.

Finally, the HIV example can be easily solved in the following way.

(3') $((Fx \wedge Hx) \rightarrow Gx?) \not\subset (\neg Gx \vee Ax)$

(8) $((Ax \wedge Ix) \rightarrow \neg Gx?) \not\subset Gx$

(9) $\forall x(Fx \rightarrow Ix)$

(10) $Ab \wedge Hb \wedge Fb$

where A stands for having contracted HIV and I for having an infection. The solution here is similar to our first example. Since (7) has priority over (3'), we will be able to conclude only $\neg Gb?$ and consequently the ambiguity will not arise.

6. Conclusion

My main objective in this paper was to show some of the promising advantages of a qualitative approach to the intra-theoretical scientific inferences. By analyzing Hempel's I-S model, we intended to show the weaknesses and unsolved problems of the best-developed formalization of the deductive-probabilistic view of intra-theoretical scientific inferences. By presenting a logical system of plausibility, we showed how a qualitative approach could solve the main problems presented by Hempel's model when considered as a model for representing intra-theoretical scientific inferences.

Besides the use of the qualitative concept of plausibility, our solution to the problem of inductive ambiguities emphasized two main points. The first one is related to use of the notion of exception. Since this notion is a stronger type of specificity, it showed to be able to solve the class specificity cases of ambiguities analyzed by Hempel as well as other cases that his model was not able to solve. Secondly, since we find the rejection of arguments and the consequent loss of information a weakness (and, as we hope to have showed, an unnecessary approach), we do not follow Hempel's approach of rejecting contradictory arguments. By the use of the exception mechanism it was possible to accept the arguments rejected by Hempel's RMS and at the same time to prevent their contradictory conclusions. In the cases where there is no way to prevent the ambiguity, that is, when there is a real "two-all draw", our approach allows for a paraconsistent treatment that accepts the ambiguity and consequently prevents the rejection of arguments and the loss of information. While elegantly solving the cases that Hempel's model was not able to do, our solution simply keeps the contradictions and paraconsistently allows further inferences upon them. Since the problem of ambiguities was solved without imposing, like Hempel's RMS does, a "quasi-omniscient" requirement upon our weak ability to know, at first glance our approach seems to be useful in practical applications of intra-theoretical scientific inferences.

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