Qualitative Logic for 'Generally'

Sheila R. M. Veloso and Paulo A. S. Veloso
Systems and Computer Engin. Program, COPPE-UFRJ, Brazil;
Computer Sci. Dept., Inst. Mathematics, UFRJ, Brazil.

Abstract. We examine logical systems with generalized quantifiers, for expressing and reasoning about assertions with 'generally'. The primary motivation is a qualitative, rather than quantitative, approach to some vague notions. Assertions and arguments involving notions, such as 'generally', 'most', 'many', etc., occur often in ordinary language and in some branches of science. Filters can be used for capturing an intuitive idea of 'generally'. This motivates the introduction of an operator to express 'generalized' assertions whose meaning is intended to be "belonging to a given filter (of 'important' sets)". These ideas are incorporated in a basic (unsorted) logic. Some interesting situations may require assertions relative to several universes, involving "most birds" and "most penguins" for instance. This leads to our sorted framework for reasoning about versions of 'generally' relative to various universes.

Keywords: Logic, generalized quantifiers, vague notions, 'generally', sorted framework.

1 Introduction

We examine logical systems with generalized quantifiers, for expressing and reasoning about 'generally'. Assertions and arguments involving vague notions, such as 'generally', 'most', 'many', etc., occur often both in ordinary language and in some branches of science. The primary motivation is a qualitative, rather than quantitative, approach to such vague notions.

We wish to express assertions, such as "birds 'generally' fly", and reason about them in a formal manner. One usually understands "birds 'generally' fly" as "all birds, but for a 'negligible' set of exceptions, fly", which suggests the paraphrases "most birds fly" or "the set of flying birds is an 'important' set". To express such 'generally' assertions formally, we introduce the new operator and express birds 'generally' fly" by vF(v). To give precise meaning to such assertions, we extend the usual notions by providing a family $\mathcal{F}$ of 'important' sets and stipulate that vF(v) is to mean "the set { b B : F(v) } of flying birds is in the family $\mathcal{F}$" as a rigorous counterpart for "the set of flying birds is an 'important' set". To reason about such 'generalized' assertions in a formal manner, we will set up a deductive system by extending (conservatively) the classical first-order predicate calculus. This logic is related to default logic [1] and its variants [2, 3], as indicated by benchmark examples, which was one of motivations for the introduction of some related systems [4, 5]. But, they are quite different logical systems, both technically and in terms of intended interpretation [6, 7].

In this paper, we consider 'generally' in the sense of 'most' and examine a logic with generalized quantifiers on filters similar to ultrafilter logic [6, 8]. The aim is indicating how some ideas and results about ultrafilter logic can be adapted to filter logic. The structure of this paper is as follows. We begin by motivating the usage of filters for capturing an intuitive idea of 'generally'. Next, we consider a basic (unsorted) framework: we introduce our logic for 'generally', in section 3, and examine its properties, such as completeness, in section 4. Some interesting situations may require assertions relative to several universes, involving "most birds" and "most penguins" for instance, which we take up in section 5, where we motivate ideas concerning 'relative generally' and introduce our sorted framework for reasoning about versions of 'generally' relative to various universes.

2 On 'Generally' and 'Negligible'

We will now indicate how one can arrive at filters [9] as capturing an idea of 'generally'. The approach is based on the familiar intuition of 'generally' as "all but for a 'negligible' set of exceptions", but we also employ some related, and more basic notions [10].

We will first motivate and outline our approach to making precise a notion of 'generally'. Since we shall be dealing with local qualitative notions, we will prefer to use names like 'important' and 'negligible'. In the sequel, we will indicate that filters [9] are appropriate for giving precise counterparts for such notions. The intuition of 'generally' as "all but for a 'negligible' set of exceptions" suggests understanding "objects 'generally' have a given property" as "the exceptional objects (failing to have this property) form a 'negligible' set. If we understand 'negligible' as "fit to be neglected or discarded", it appears reasonable to say that two sets are 'almost as important' when their difference (i. e. the part where they differ) is negligible. The difference is the so-called symmetric difference: XΔY:=(X-Y)≈(Y-X).

We can now put forward some postulates about these notions (based on common sense and ordinary understanding), namely "Sets with negligible symmetric difference are almost as important"; "Subsets of negligible sets are negligible"; "Sets almost as important as negligible ones are negligible"; "The empty set is negligible"; and "The universe V is not negligible" [10].

In virtue of these five postulates, the family of negligible sets forms a proper ideal [9]. Conversely, each proper ideal is a family of subsets satisfying our five postulates. Thus, the interpretation of "objects 'generally' have a given property φ" as "the set of objects failing to have property φ is negligible" can be seen to amount to "the set of objects having φ belongs to a given filter" (the family $\mathcal{F}$ of sets with negligible complements).

3 A Logic for 'Generally'

Our logic for 'generally' adds to classical first-order logic a generalized quantifier for expressing it. We now examine this logic $\mathcal{L}_{\omega\omega}(\mathcal{F})$: its syntax, semantics and axiomatics.

Given a signature ρ, we use L(ρ) for the usual first-order language (with equality +) of signature ρ, and L (ρ) for the extension of L(ρ) by the new operator . The formulas of L (ρ) are built by the usual formation rules and the new variable-binding formation rule for generalized formulas: for each variable v, if φ is a formula then so is vφ. Other syntactic notions, such as substitution (φ(t)), can be easily adapted [11, 12].

To illustrate the expressive power of such languages with , consider a signature with a binary predicate L (standing for 'loves'). Some assertions expressed by means of are: "people generally love somebody" by x∃yL(x,y), "somebody loves people in general" by ∃x yL(x,y), "people generally love everybody" by x∀yL(x,y), and "people generally love each other" (in the sense "most people love most people") by x yL(x,y).

The semantic interpretation of our logic $\mathcal{L}_{\omega\omega}(\mathcal{F})$ is provided by enriching first-order structures with filters and extending the definition of satisfaction to the new quantifier .

A *filter structure* $A^{\mathcal{F}} = (A, \mathcal{F})$ for signature ρ consists of a usual structure A for ρ together with a filter $\mathcal{F}$ over the universe A of A. We extend the Tarskian definition of *satisfaction* of a formula in a structure under assignment a to its (free) variables as follows: we define $A^{\mathcal{F}}$ p vφ(u,v) [a] iff the set { b A : $A^{\mathcal{F}}$ p φ(u,v) [a,b] } is in $\mathcal{F}$.

Satisfaction of a formula hinges only on the realizations assigned to its symbols. Other semantic notions, such as reduct and model ($A^{\mathcal{F}}$ p Γ), are as usual [11, 12]; also the notion of *filter consequence* is as expected: Γ $p^{\mathcal{F}}$ τ iff $A^{\mathcal{F}}$ p τ whenever $A^{\mathcal{F}}$ p Γ (likewise for validity).

We will now formulate a deductive system for our logic by adding schemata (coding properties of filters) to a calculus for classical first-order logic. We set up a deductive system for filter logic by taking a sound and complete deductive calculus for classical first-order logic, with Modus Ponens (MP) as the sole inference rule (as in [11]), and extending its set A(ρ) of axiom schemata by adding a set $\Phi^f(\rho)$ of new axiom schemata to form $A^f(\rho) := A(\rho) \approx \Phi^f(\rho)$. This set $\Phi^f(\rho)$ consists of all the generalizations of the following five schemata (where φ, ψ and θ are formulas of L (ρ)):

[∀]: ∀vφ∅ vφ; [∃]: vφ∅∃vφ;
[ ]: (vψ vθ)∅ v(ψ θ);
[∅]: ∀v(ψ∅θ)∅(vψ∅ zθ);
[v]: vφ(v)∅ uφ(u), for a new variable u.

These schemata express properties of filters, with [v] covering alphabetic variants. Other usual deductive notions, such as (maximal) consistent sets, witnesses and conservative extension [11, 12], can be easily adapted. Filter derivations are first-order derivations from the filter schemata: $\Gamma \circ^f \varphi$ iff $\Gamma \approx A^f(\rho) \circ \varphi$. Hence, we have monotonicity, and substitutivity of equivalents.

As an example, consider the following facts about a universe of people: "people generally oppose those in conflict with any one with whom they sympathize", expressed by the sentence $x\forall y\ z[(S(x,y)\ C(z,y))\varnothing C(x,z)]$, and "people generally sympathize with Bill", expressed by $xS(x,b)$. Then, one can infer the sentence $x\ z[C(z,b)\varnothing C(x,z)]$, i. e. "people generally oppose those in conflict with Bill".

4 Filter Logic

We shall now establish some properties of filter logic, including soundness and completeness of the deductive system with respect to filter consequence.

Our deductive system provides a sound and complete deductive calculus for reasoning about assertions involving 'generally': $\Gamma \circ^f \tau$ iff $\Gamma\, p^{\mathcal{F}} \tau$. Soundness ($\circ^f \prod p^{\mathcal{F}}$) is easily established as usual. For completeness ($p^{\mathcal{F}} \prod \circ^f$), we can adapt Henkin's well-known proof for classical first-order logic [11, 12, 13], by providing an adequate filter by means of witnesses. We proceed to outline how this can be done (cf. [6, 8]).

Given a consistent set Γ in $L(\rho)$, extend it to a maximal consistent set Σ in $L(\rho^*)$, with witnesses in set C of new constants for the existential sentences of $L(\rho^*)$, where $\rho^* := \rho \approx C$. We form the canonical structure H, for signature ρ^*, with universe H, as usual, and provide a filter, by means of the formulas of language $L(\rho^*)$ with a single free variable, as follows. Considering the set represented within Σ by formula $\varphi(v)$ of $L(\rho^*)$, namely $\varphi(v)^\Sigma := \{ t/0^\Sigma\ H : \varphi(t)\ \ \Sigma \}$, we form the family of provably important subsets of H: $\Sigma := \{ \varphi(v)^\Sigma \prod H :\ v\varphi(v)\ \ \Sigma \}$. By our axioms, this family Σ has the finite intersection property, so its closure $\mathcal{F}_\Sigma \prod\ (H)$ under supersets is a filter. We use this filter $\mathcal{F}_\Sigma$ on H to expand the canonical structure H to a filter structure $H^{\mathcal{F}_\Sigma} := (H, \mathcal{F}_\Sigma)$ for ρ^*. We can now show, by induction, that $H^{\mathcal{F}_\Sigma}\, p\, \tau$ iff $\tau\ \ \Sigma$, for each sentence τ of $L(\rho^*)$. The inductive step for the new quantifier , namely, for a sentence $v\varphi$ ($H^{\mathcal{F}_\Sigma}\, p\ \ v\varphi$ iff $v\varphi\ \ \Sigma$) follows from the crucial property $\varphi(v)^\Sigma\ \ \Sigma$ iff $\varphi(v)^\Sigma\ \ \mathcal{F}_\Sigma$ (due to schema [$\varnothing$]).

We thus have a Löwenheim-Skolem Theorem for our filter logic $\mathcal{L}_{\omega\omega}(\mathcal{F})$.

Löwenheim-Skolem Theorem for filter logic. Each $\circ^f$-consistent set Γ of sentences of $L(\rho)$ has a filter model $M^{\mathcal{F}}$ with cardinality at most that of the language: $|M| \leq |L(\rho)|$.

Hence, we have the desired completeness result for our filter logic.

Theorem. The deductive system $\circ^f$ is complete with respect to the consequence $p^{\mathcal{F}}$: $\Gamma \circ^f \tau$ whenever $\Gamma\, p^{\mathcal{F}} \tau$.

We now examine other metamathematical properties of our filter logic $\mathcal{L}_{\omega\omega}(\mathcal{F})$ for 'generally'. We have a sound and complete deductive system for $\mathcal{L}_{\omega\omega}(\mathcal{F})$. As usual, such a result transfers the finitary character of $\circ^f$ to the compactness of $p^{\mathcal{F}}$. Thus, our logic is a proper extension of classical first-order logic with compactness and Löwenheim-Skolem properties. Also, $\mathcal{L}_{\omega\omega}(\mathcal{F})$ has some other connections with classical first-order logic $\mathcal{L}_{\omega\omega}$: its conservativeness over $\mathcal{L}_{\omega\omega}$ and the universality of -consequences of first-order theories.

Proposition. Consider a set Δ of sentences and a formula θ of $L(\rho)$.

a) Conservativeness of $\mathcal{L}_{\omega\omega}(\mathcal{F})$ over $\mathcal{L}_{\omega\omega}$: $\Delta \circ \theta$ iff $\Delta \circ^f \theta$.

d) Generalized consequences: $\Delta \circ^f\ v\theta$ iff $\Delta \circ \forall v\theta$ and $\Delta \circ^f\ \ v\theta$ iff $\Delta \circ\ \exists v\theta$.

Proof outline. Any nonempty set can be extended to some proper filter.

Item (b) corroborates that 'generally' requires explicit information, otherwise it reduces to classical quantification (only the universe can be guaranteed to be in every filter).

Example. Consider consistent theories with information about a universe of birds.

a) Consider a consistent purely first-order theory. Assume that one knows that "some birds fly", "every bird is a biped with beak", and "flying birds have wings". Then, one does not know that "birds generally do not fly", i. e. $\Delta O^f\ v\ \ F(v)$. Also, not knowing that "all birds fly"

($\Delta O \forall v F(v)$), one does not know that "birds generally fly" ($\Delta O^f \; vF(v)$).

b) Consider a consistent theory Γ with generalized information. Assume that, besides "all feathered winged birds fly", one knows that "birds generally have wings" and "birds generally have feathers". Then, one concludes that "birds generally fly": $\Gamma o^f \; vF(v)$).

5 Relative 'Generally'

We shall now examine the idea of having a notion of 'generally' relative to a universe: how it arises and can be formulated as well as some related issues (cf. [8]). We will first indicate how the proper expression of "relative generally" assertions brings about the idea of a notion of important with respect to each universe, leading to its natural formulation in a sorted version of filter logic. Then, the need for establishing some connections while blocking others leads to comparing such relative concepts. Finally, these ideas will be incorporated into a sorted framework for reasoning about relative generally.

5.1 Basic Ideas

Our generalized quantifier may exhibit somewhat unexpected behavior in some cases. We shall now examine these undesirable side-effects and propose a way to overcome this difficulty.

The generalized quantifier is meant to capture the idea of holding generally, i. e. for 'most' of objects of the universe. Sometimes we wish to express the idea of holding generally over a given subset of the universe, i. e. for 'most' objects of a given sub-universe. We now examine the expression of such relative generally assertions.

Over a universe B of birds, we express "birds generally fly" by $vF(v)$. How are we to express relative generalized assertions, like "eagles generally have wings" or "penguins generally have beaks"? By analogy with the classical quantifiers, relativization is an apparently natural suggestion: express "M's generally are N's" by $v[M(v)\varnothing N(v)]$. Unfortunately, relativization fails to be adequate for expressing 'relative generalized' assertions, due to the behavior of the quantifier .

As an example to illustrate this issue, consider expressing some facts about birds by relativization: "all penguins are winged birds" by $\forall v[P(v)\varnothing W(v)]$, and "penguins generally do not fly" by $v[P(v)\varnothing \; F(v)]$. From these two sentences, one concludes $v[W(v)\varnothing \; F(v)]$, which would be read as "winged birds generally do not fly". Now, the two given premises appear to express reasonable facts. On the other hand, the conclusion, <u>as read</u>, does not look so reasonable. This example indicates that relativization fails to express the intended idea. The reason comes from neglecting the relative aspect.

For a formula $v[M(v)\varnothing N(v)]$ the reading "M's generally are N's" is not appropriate. For, one must bear in mind that what this does assert is "for most <u>objects</u> a, if M(a) then N(a)". A natural approach to overcome this problem, thus expressing 'relative generally', rests on relative notions of 'important': each given universe has its own relative notion of 'important' subsets. This idea may be formulated by providing a proper filter $\mathcal{F}_S$ over each universe S. With such relative notions of 'important', we can paraphrase "M's generally are N's" as "most M's are N's" meaning that the set $\{a \;\; M : N(a)\}$ is 'almost as important as' the universe M, i. e. $M \leftrightarrow N \spadesuit M$ or $M \leftrightarrow N \;\; \mathcal{F}_M$.

A many-sorted approach can provide a framework for formulating the idea of distinct notions of 'important' for the universes, where one assigns proper families corresponding to these relative notions of important. We shall now examine sorted versions of our logics for 'generally'. The basic idea is relativizing to sorts the previous (unsorted) concepts.

We consider many-sorted signatures, where the extra-logical symbols, as well as variables, come classified according to sorts [11]. Quantifiers are relativized to sorts, as expressed in the formation rules: for each variable v over sort s, if φ is a formula in L (ρ), then so are $(\forall v{:}s)\varphi$, $(\exists v{:}s)\varphi$ and $(\;\; v{:}s)\varphi$. A *filter structure* $A^{\mathcal{F}}$ for S-sorted signature ρ is an expansion of an S-sorted (first-order) structure A for ρ, obtained by assigning to each sort s of signature ρ a filter $\mathcal{F}_s$ over the universe A[s] of sort s (giving the important subsets of A[s]). The definition of *satisfaction* becomes relativized to sorts accordingly: $A^{\mathcal{F}} p \;\; v\varphi(\underline{u},v)\,[\underline{a}]$ iff the set $\{b \;\; A[s] : A^{\mathcal{F}} p\, \varphi(\underline{u},v)\,[\underline{a},b]\}$ is in $\mathcal{F}_s$. The filter *axiom schemata* in the set $\Phi^f(\rho)$ become sorted as well

As in classical first-order logic, the sorted and unsorted versions are quite similar. So, the results in section 4 (e. g. soundness and completeness) carry over to the sorted version, by relativizing to sorts the previous arguments.

5.2 Sorted Framework

We now take a closer look at the proposal of employing distinct notions of important subsets.

We will examine how the need for establishing some connections while blocking others leads to comparing these relative notions of important sets.

The next example shows how some (undesired) conclusions can be blocked.

Example. Given that "All penguins are birds" ($P \prod B$), consider the assertions σ:"birds generally fly" (the flying birds form an important set of birds, i. e. $B \leftrightarrow F \ \ \mathcal{F}_B$) and τ:"penguins generally fly" (the flying penguins form an important set of penguins, i. e. $P \leftrightarrow F \ \ \mathcal{F}_P$). Now, neither σ entails τ (since we may even have $P \leftrightarrow F =$), nor does τ entail σ (since $P \prod B$ may very well be a negligible set of <u>birds</u>), if the relative notions of important sets are not connected.

This example illustrates the idea of independent notions of important subsets. If the set of penguins is not an important set of <u>birds</u> ($P \prod B$ not almost as important as B), then a set $X \prod P$ (e. g., of non-flying penguins) may be an important set of <u>penguins</u> without being an important set of <u>birds</u>.

The next example shows how some (desired) conclusions can be achieved.

Example. Given that "All winged birds are birds" ($W \prod B$), consider the assertions σ:"birds generally fly" (as before $B \leftrightarrow F \ \ \mathcal{F}_B$ or $B \leftrightarrow F \spadesuit B$) and μ:"winged birds generally fly" (the flying winged birds form an important set of winged birds, i. e. $W \leftrightarrow F \ \ \mathcal{F}_W$ or $W \leftrightarrow F \spadesuit W$). Given also that "birds generally have wings" ($W \spadesuit B$), the set $B - W$ of exceptional wingless birds is a negligible set (of <u>birds</u>). So, it appears intuitively plausible that the important subsets of W are the relativizations $W \leftrightarrow Y$ of the important subsets Y of B. Thus, we shall also assume the coherence principle: for any subset $Y \prod B$, $W \leftrightarrow Y \spadesuit W$ iff $Y \spadesuit B$. In the presence of this principle, assertions σ and μ become equivalent.

The two preceding examples illustrate the following ideas. Given $S \prod T$ and a proper filter $\mathcal{F}_S$ over S, consider the *relativizable complex* ${}^T\mathcal{F}_S := \{ Y \prod T : S \leftrightarrow Y \ \ \mathcal{F}_S \}$. If S $\mathcal{F}_T$ (say because $(T - S)$ $\mathcal{F}_T$), then we need an independent notion $\mathcal{F}_T$ of important subsets of T. If S $\mathcal{F}_T$, then relativizable complex ${}^T\mathcal{F}_S$ is a filter over S, which we may take as $\mathcal{F}_T$, if we wish to enforce coherence inheritance: for every subset $Y \prod T$, $S \leftrightarrow Y \ \ \mathcal{F}_S$ iff Y $\mathcal{F}_T$.

We shall now consider comparison of universes, with distinct notions of important subsets, in a sorted framework. We shall examine how to formulate some ideas related to sub-universes and coherent inheritance in a many-sorted approach.

In our sorted framework, sorts are unrelated: we have equality only over a sort, rather than between distinct sorts. Nevertheless, we can express some relationships among sorts by means of appropriate injections. The idea is that an injection i from s to t establishes a bijection from its domain s onto its image i[s], the latter being a subset of t. To express that s is a subsort of t, we resort to a unary function i from s to t together with an axiom asserting its injectivity [14]. This yields transitivity of subsorts.

We now formulate the previous coherence inheritance principle for an injection $i : s \varnothing t$, where the image i[s] is an important subset of t. Then, the non-image $t - i[s]$ is a negligible subset of t, where the distinction between a set $Z \prod t$ and its pre-image $i^{-1}[Z]$ is confined. So, we may consider $Z \prod t$ as an important subset ($Z \ \ \mathcal{F}_t$) of t iff $i^{-1}[Z]$ is an important subset of s ($i^{-1}[Z] \ \ \mathcal{F}_s$).

Now, given $i : s \varnothing t$ and a formula $\varphi(z)$ with variable z over t, we can express "objects of t generally are in the image" by the sentence χ_i: $(\ z{:}t)(\exists x{:}s) z{+}i(x)$, express "objects of t generally have property φ" by φ_t: $(\ z{:}t)\varphi(z)$, and use φ_s: $(\ x{:}s)\varphi(i(x))$ to express "objects of s generally give objects in t with property φ". This leads to the *coherent inheritance schema* $[\ \ i{:}s \prod t]$, with instances $(\ \ i{:}s \prod t/\varphi)$ as $\chi_i \varnothing (\varphi_t \times \varphi_s)$.

Let us examine our preceding examples in this sorted formulation.

Example. Consider three sorts: b (for birds), w (for winged birds) and p (for penguins) and a unary predicate F (for flies) over sort b, with j:w∅b and k:p∅b.

a) Considering all winged birds as birds, assume (z:b)(∃x:w)z+j(x) {"birds generally have wings"}. Then, instance (j:w∏b/F(z)) of the inheritance schema yields the equivalence between (z:b)F(z) {"birds generally fly"} and (x:w)F(j(x)) {"winged birds generally fly"}. We thus see that, as the winged birds form an important set of birds, "generally flying" is inherited both downwards and upwards.

b) Considering all penguins as birds, assume (z:b)F(z) {"birds generally fly"}. Now, if we have (z:b)(∃y:p)z+k(y) {"birds generally are penguins"}, instance (k:p∏b/F(z)) will yield (y:p)F(k(y)) {"penguins generally fly"}; but otherwise this conclusion is not forced upon us. In fact, from the sentence (x:p) F(k(y)) {"penguins generally do not fly"}, the instance (k:p∏b/ F(z)) of the schema can be seen to yield (z:b)(∃y:p)z+k(y) {"it is not the case that birds generally are penguins"}.

This example illustrates how the coherence inheritance schema provides uniform control based on the relative importance of the sorts.

6 Conclusion

We have examined monotonic logical systems with generalized quantifiers over filters, which provide rigorous bases for qualitative reasoning with vague notions, such as 'generally' in the sense of 'most'. The unsorted logical system is a conservative extension of classical first-order logic, with which it shares several properties. Some situations, however, require assertions relative to several universes, leading to the idea of 'relative generally' and our sorted framework for them. We can similarly introduce generalized quantifiers for the dual notion of 'negligible'. Modal versions of these logics can also be contemplated.

This logical system, though related to default logics, are quite different, both technically and in terms of intended interpretation [7]. Our filter logic belongs to a family of closely related systems with generalized quantifiers for qualitative reasoning about vague notions, including ultrafilter logic [6, 8, 15]. These systems appear to have interesting connection with fuzzy logic [16, 17] (e. g. expressing 'very tall' by "taller than most"), as well as with empirical reasoning [18], which suggest the possibility of other applications [6, 8]. They are undergoing further investigation [19, 20]. For instance, one can adapt the natural deduction system for ultrafilter logic [21] to our filter logic and similar ones.

7 Acknowledgments

The authors gratefully acknowledge partial financial support from the Brazilian National Research Council (CNPq): grants 301163/91-0 (to PASV) and 522979/96-3 (to SRMV), at IM and COPPE, UFRJ.

8 References

[1] Reiter, R. A logic for default reasoning. *J. Artificial Intelligence* 13: 81-132, 1980.

[2] Antoniou, G. *Nonmonotonic Reasoning*. MIT Press, 1997.

[3] Brewka, G., Dix, J. and Konolige, K. *Nonmonotonic Reasoning: an overview*. CSLI, Stanford, 1997.

[4] Carnielli, W. A. and Sette, A. M. Default operators. *Abstr. Workshop on Logic, Language, Information and Computation*, Recife, 1994.

[5] Schlechta, K. Defaults as generalized quantifiers. *J. Logic and Computation* 5: 473-494, 1995.

[6] Carnielli, W. A. and Veloso, P. A. S. Ultrafilter logic and generic reasoning. In Gottlob, G., Leitsch, A., and Mundici, D. (eds.) *Computational Logic and Proof Theory* {LNCS 1289}: 34-53, Springer-Verlag , 1997.

[7] Sette, A. M., Carnielli, W. A. and Veloso, P. A. S. An alternative view of default reasoning and its logic. In Haeusler, E. H. and Pereira, L. C. (eds.) *Pratica: Proofs, Types and Categories*:127-158, PUC-Rio, 1999.

[8] Veloso, P. A. S. On ultrafilter logic as a logic for 'almost all' and 'generic' reasoning. Res. Rept. ES-488/98, COPPE-UFRJ, 1998.

[9] Bell, J. L. and Slomson, A. B. *Models and Ultraproducts: an introduction*, North-Holland, 1971.

[10] Veloso, P. A. S. On 'almost all' and some presuppositions. In Pereira, L. C. P. D. and

Wrigley, M. B. (eds.) *Logic, Language and Knowledge: essays in honour of Oswaldo Chateaubriand Filho*, Manuscrito XXII: 469-505, 1999.

[11] Enderton, H. B. *A Mathematical Introduction to Logic*. Academic, 1972.

[12] Shoenfield, J. R. *Mathematical Logic*. Addison-Wesley, 1967.

[13] Henkin, L. The completeness of the first-order functional calculus. *J. Symbolic Logic* 14: 159-166, 1949.

[14] Meré, M. C. and Veloso, P. A. S. Definition-like extensions by sorts. *Bull. IGPL* 3: 579-595, 1995.

[15] Grácio, M. C. G. *Lógicas Moduladas e Raciocínio sob Incerteza*, D. Sc. dissertation, UNICAMP, Campinas, 1999.

[16] Turner, W. *Logics for Artificial Intelligence*. Ellis Horwood, 1984.

[17] Zadeh, L. A. Fuzzy logic and approximate reasoning. *Synthèse* 30: 407-428, 1975.

[18] Hempel, C. *Aspects of Scientific Explanation and Other Essays in the Philosophy of Science*. Free Press, 1965.

[19] Veloso, P. A. S. and Carnielli, W. A. Logics for Qualitative Reasoning. CLE e-Prints 1 2001, available at http://www.cle.unicamp.br/e-prints.

[20] Veloso, P. A. S. On interpolation and modularity for ultrafilter logic. In Abe, J. M. and Silva Filho, J. I. (eds.) *Logic, Artificial Intelligence and Robotics*: 270-278, IOS Press, 2001.

[21] Rentería, C. J., Haeusler, E. H. and Veloso, P. A. S. NUL- Natural Deduction for Ultrafilter Logic. In Natural Deduction 2001, Rio de Janeiro, 2001.